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W—71—2018

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

Paper IX

(Real Analysis-II)

(MCQ+Theory)

(Tuesday, 16-10-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes are for Q. No. **1** (MCQ) and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken circle of correct choice in OMR answer-sheet.

(*v*) Negative marking system is applicable for MCQ.

(MCQ)

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) Which of the following is *true* ?

(A) Every real valued bounded monotonic function is Riemann integrable

(B) Every bounded function is Riemann integrable

(C) Every Riemann integrable function is continuous

(D) Every Riemann integrable function is continuous except for finite number of points

(*ii*) Let a function $f : [0, 1] \to \mathbf{R}$ be defined by :

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ 0, & \text{when } x \text{ is irrational} \end{cases}$$

Then :

(A) f is Riemann integrable on [0, 1]

(B) f is continuous on [0, 1]

(C) f is not integrable on [0, 1]

(D) f is monotonic function on [0, 1]

P.T.O.

(*iii*) If P* is a refinement of partition P of $[a, b]$, then for a bounded function f on $[a, b]$:

(A) $L(P, f) \leq L(P^*, f)$

(B) $U(P^*, f) \leq U(P, f)$

(C) $U(P^*, f) - L(P^*, f) \leq U(P, f) - L(P, f)$

(D) All of the above

(*iv*) $\int_{-1}^{1} |x| \,.\, dx =$

(A) -1 (B) 0

(C) 1 (D) $\frac{1}{2}$

(*v*) Which of the following is *not* improper integral ?

(A) $\int_{0}^{1} \frac{1}{x} dx$ (B) $\int_{1}^{2} \frac{1}{x} dx$

(C) $\int_{1}^{\infty} \frac{1}{x} \,.\, dx$ (D) $\int_{-\infty}^{\infty} \frac{1}{x} \,.\, dx$

(*vi*) The primitive of x^2 is :

(A) $\frac{x^3}{3}$ (B) $2x$

(C) $2x + 3$ (D) x^3

(*vii*) Consider the following statements :

(*i*) The improper integral $\int_{a}^{b} f \,.\, dx$ is convergent

(*ii*) The integral $\int_{a}^{b} |f| \,.\, dx$ is convergent, then :

(A) (*i*) $\Rightarrow$ (*ii*) (B) (*ii*) $\Rightarrow$ (*i*)

(C) (*i*) $\Leftrightarrow$ (*ii*) (D) none of these (*viii*) A function $f(x) = e^x$ is :

(A) even (B) odd

(C) both even and odd (D) neither even nor odd

(*ix*) If $f(x)$ is an even function which is bounded and integrable on $[-\pi, \pi]$, then a_n =

(A) $\frac{1}{\pi}.\int_0^{\pi} f(x) . \cos nx . dx$ (B) $\frac{2}{\pi}.\int_0^{\pi} f(x) . \cos nx . dx$

(C) 0 (D) $\frac{2}{\pi}\int_0^{2\pi} f(x) . \cos nx . dx$

(*x*) Which of the following is *true* ?

(A) every power series is Fourier series

(B) every trigonometric series is Fourier series

(C) every Fourier series is trigonometric series

(D) none of the above

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) Prove that the oscillation of a bounded function f on an interval $[a, b]$ is the supremum of the set $\{|f(x_1) - f(x_2)| : x_1, x_2 \in [a, b]\}$ of numbers.

(*b*) If a function f is monotonic on $[a, b]$, then prove that f is integrable on $[a, b]$.

(*c*) Show that the constant function k is integrable and :

$$\int_a^b k.\, dx = k(b - a).$$

3. Attempt any *two* of the following : 5 each

(*a*) If a function f is bounded and integrable on $[a, b]$ and there exists a function F s.t. $F' = f$, then prove that :

$$\int_a^b f(x) .\, dx = F(b) - F(a).$$

P.T.O.

(*b*) Prove that the improper integral $\int_a^b \frac{1}{(x-a)^n}\,dx$ converges if and only if $n < 1$.

(*c*) Test the convergence of :

$$\int_0^1 \frac{dx}{\sqrt{1-x^3}}.$$

4. Attempt any *two* of the following : 5 each

(*a*) For a periodic function of period 2π, prove that :

(*i*) $\int_{\alpha}^{\beta} f \,.\, dx = \int_{\alpha+2\pi}^{\beta+2\pi} f \,.\, dx$

(*ii*) $\int_{-\pi}^{\pi} f \,.\, dx = \int_{\alpha}^{\alpha+2\pi} f \,.\, dx$

where α, β being any numbers.

(*b*) Explain the half range sine series.

(*c*) Find the Fourier series of the periodic function f with period 2π, defined as follows :

$$f(x) = \begin{cases} 0, & -\pi \le \pi \le 0 \\ x, & 0 \le x \le \pi \end{cases}.$$