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W—86—2018

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

Paper X

(Ring Theory)

(MCQ + Theory)

(Friday, 19-10-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Question No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball point pen to darken the circle of OMR sheet for Question No. **1**.

(*v*) Negative marks system is applicable for Question No. **1**.

MCQ

1. Choose the correct alternative for each of the following ; 1 mark each

(1) Let R be the set of integers and f is the usual addition, is the usual multiplication of integers. Then :

(A) R is a commutative ring with unit element

(B) R is a commutative ring but has no unit element

(C) R is not a commutative ring

(D) R is not a ring

P.T.O.

(2) If R is the set of integers mod 7 under the addition and multiplication mod 7, then :

(A) $\bar{1}.\bar{1} = \bar{1}$ (B) $\bar{2}.\bar{4} = \bar{1}$

(C) $\bar{3}.\bar{5} = \bar{1}$ (D) All of the above are true

(3) Let R be a ring, $R' = R$ and define $\phi(x) = x$ for every $x \in R$. Then :

(A) ϕ is not homomorphism

(B) ϕ is homomorphism and $I(\phi) = R$

(C) ϕ is homomorphism and $I(\phi)$ consists only of 0

(D) None of the above

(4) An ideal $M \neq R$ in a ring R is said to be a maximal ideal of R if whenever U is an ideal of R such that $M \subset U \subset R$, then :

(A) R = U (B) M = U

(C) $R \neq U$ (D) Either R = U or M = U

(5) An integral domain is said to be a Euclidean ring if for every $a \neq 0$ in R there is defined a non-negative integer $d(a)$ such that :

(A) For all $a, b \in R$, both non-zero $d(a) \leq d(ab)$

(B) For any $a, b \in R$ both non-zero there exists $t, r \in R$ such that $a = tb + r$, where either $r = 0$ or $d(r) < d(b)$

(C) Only condition (A) is needed

(D) Both conditions (A) and (B) are needed

(6) If $x \neq 0 \in J[i]$, then :

(A) $d(x) \leq 1$ (B) $d(x) \geq 1$

(C) $d(x) < 1$ (D) $d(x) = 1$

(7) If p is a prime number of the form $4n+1$, then :

(A) $p = a^2 + b^2$ for some integers a, b

(B) $p = a^2 b^2$ for some integers a, b

(C) $p = a^2 + b^2$ for all integers a, b

(D) $p = a^2 b^2$ for all integers a, b

(8) If $f(x)$, $g(x)$ are non-zero elements in F[x], then :

(A) $\deg f(x) \geq \deg f(x)\, g(x)$

(B) $\deg f(x) \leq \deg f(x)\, g(x)$

(C) $\deg f(x) < \deg f(x) + \deg g(x)$

(D) $\deg f(x) > \deg f(x)\, g(x)$

(9) Consider two statements :

(I) A finite integral domain is a field

(II) Every field is an integral domain.

Then :

(A) Both statements (I) and (II) are false

(B) Both statements (I) and (II) are true

(C) Only statement (I) is true

(D) Only statement (II) is true

(10) The polynomial x^2+1 is :

(A) irreducible over the real field and complex field

(B) irreducible over real field but not over the complex field

(C) not irreducible over both real and complex field

(D) None of the above

Theory

2. Attempt any *two* of the following : 5 marks each

(*a*) If R is a ring, then for all $a, b \in \mathrm{R}$, prove that :

(*i*) $a.0 = 0.a = 0$

(*ii*) $(-a)\,(-b) = ab$

(*b*) If ϕ is a homomorphism of R into R$'$, then prove that :

(*i*) $\phi(0) = 0$

(*ii*) $\phi(-a) = -\phi(a)$ for every $a \in \mathrm{R}$

(*c*) Let $\mathrm{J}[\sqrt{2}]$ be all real numbers of the form $m + n\sqrt{2}$ where m, n are integers, then verify that $\mathrm{J}[\sqrt{2}]$ forms a ring under usual addition and multiplication of real numbers.

P.T.O.

3. Attempt any *two* of the following : 5 marks each

(*a*) If U is an ideal of the ring R, then prove that R/U is a ring.

(*b*) If U is an ideal of ring R and $1 \in U$, then prove that U = R.

(*c*) Let R be a Euclidean ring. Then prove that any two elements a and b in R have a greatest common divisor d. Also prove that $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$.

4. Attempt any *two* of the following : 5 marks each

(*a*) Prove that if p is a prime number of the form $4n+1$, then we can solve the congruence $x^2 \equiv -1 \bmod p$.

(*b*) If $f(x)$, $g(x)$ are two non-zero elements of F[x], then prove that :

$$\deg(f(x)\, g(x)) = \deg f(x) + \deg g(x).$$

(*c*) Show that $x^2 + x + 1$ is irreducible over F, the field of integers mod 2.