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B—100—2019

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MARCH/APRIL 2019

(CBCS/CGPA Pattern)

MATHEMATICS

Paper X

(Ring Theory)

(MCQ & Theory)

(Friday, 29-3-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Question No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball point pen to darken the circle on OMR sheet for Question No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1**.

MCQ

1. Choose the *correct* alternative for each of the following : 1 mark each

(*i*) If R is the set of rational numbers under the usual addition and multiplication of rational numbers, then R is :

(A) A non-commutative ring

(B) A commutative ring with unit element

(C) A commutative ring without unit element

(D) Not a ring

P.T.O.

(*ii*) A commutative ring without zero divisors is called as :

(A) An integral domain (B) A division ring

(C) A field (D) Euclidean ring

(*iii*) A homomorphism of R into R′ is said to be an isomorphism, if it is :

(A) Onto mapping

(B) Not one to one mapping

(C) One to one mapping

(D) Onto but not one to one mapping

(*iv*) If R is a commutative ring and $a \in$ R, then the set

$$aR = \{ar \mid r \in R\} \text{ is :}$$

(A) Only a left ideal (B) Only a right ideal

(C) Not an ideal (D) A two-sided ideal

(*v*) A non-empty subgroup U of a ring R is said to be a two-sided ideal, if for every $u \in U$ and $r \in R$:

(A) $ur \in U$ but $ru \notin U$ (B) $ru \in U$ but $ur \notin U$

(C) $ru \in U$ and $ur \in U$ (D) $ru \notin U$ and $ur \notin U$

(*vi*) In the Euclidean ring R, a and b in R are said to be relatively prime, if :

(A) Their greatest common divisor is a unit of R

(B) Their least common multiple is a unit of R

(C) Their greatest common divisor doesn't belong to R

(D) Their greatest common divisor is prime

(*vii*) If π is a prime element in the Euclidean ring R and $\pi | ab$ where $a, b \in R$, then π divides :

(A) None of a and b (B) At least one of a or b

(C) Only one of a and b (D) a but not b

(*viii*) If $p(x) = 1 + x - x^2$ and $q(x) = 2 + x^2 + x^3$, then the coefficient of x^3 in the product $p(x).q(x)$ is :

(A) 2 (B) –1

(C) 1 (D) 3

(*ix*) The content of the polynomial $f(x) = a_0 + a_1x + \ldots\ldots + a_nx^n$, where a's are integers, is :

(A) The least common multiple of the coefficients $a_0, a_1, \ldots\ldots, a_n$

(B) The sum of the coefficients $a_0, a_1, \ldots\ldots, a_n$

(C) The greatest common divisor of the coefficients $a_0, a_1, \ldots\ldots, a_n$

(D) The product of the coefficients $a_0, a_1, \ldots\ldots, a_n$

(*x*) Which of the following polynomials is not a primitive polynomial ?

(A) $2x + 4x^2$ (B) $1 + x + x^2$

(C) $2 + 3x + x^2$ (D) $x^2 + 3x + 5$

Theory

2. Attempt any *two* of the following : 5 marks each

(*a*) If ϕ is a homomorphism of R into R′, then prove that :

(*i*) $\phi(0) = 0$

(*ii*) $\phi(-\alpha) = -\phi(a)$ for every $a \in$ R.

(*b*) If R is a ring, then for all $a, b \in$ R, prove that :

(*i*) $a.0 = 0.a = 0$

(*ii*) $a(-b) = (-a)b = -(ab)$.

(*c*) Let R be a ring, R′ = R and define $\phi : R \to R'$ by $\phi(x) = x$, then show that ϕ is a homomorphism. Also find its kernel $I(\phi)$.

P.T.O.

3. Attempt any *two* of the following : 5 marks each

(*a*) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R, if and only if R|M is a field.

(*b*) Let R be a Euclidean ring. Suppose that for $a, b, c \in$ R, $a|bc$ but $(a, b) = 1$ then, prove that $a|c$.

(*c*) In a commutative ring with unit element, prove that the relation 'a is an associate of b' is an equivalence relation.

4. Attempt any *two* of the following : 5 marks each

(*a*) If the primitive polynomial $f(x)$ can be factored as the product of two polynomials having rational coefficients, then prove that it can be factored as the product of two polynomials having integer coefficients.

(*b*) If p is a prime number of the form $4n + 1$, then prove that $p = a^2 + b^2$ for some integers a, b.

(*c*) Prove that $x^2 + 1$ is irreducible over the integers mod 7.