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B—115—2019

FACULTY OF SCIENCE

B.Sc. (Fourth Semester) EXAMINATION

MARCH/APRIL, 2019

(CBCS/CGPA Pattern)

MATHEMATICS

Paper XI

(Partial Differential Equations)

(MCQ+Theory)

(Monday, 1-4-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Use only black point pen for first question.

(*iii*) Darken only one circle for most correct answer of each MCQ.

(*iv*) Negative marking system is applicable for first question.

(MCQ)

1. Choose the most *correct* answer of the following : 10

(*i*) Which is the auxiliary equation of the equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$:

(A) $\dfrac{dx}{x^2 - yz} = \dfrac{dy}{y^2 - zx} = \dfrac{dz}{z^2 - xy}$

(B) $\dfrac{dx}{z^2 - xy} = \dfrac{dy}{y^2 - zx} = \dfrac{dz}{x^2 - yz}$

(C) $\dfrac{dx}{y^2 - zx} = \dfrac{dy}{z^2 - xy} = \dfrac{dz}{x^2 - yz}$

(D) None of the above

(*ii*) The integration with respect to x of the equation $\dfrac{\partial^2 z}{\partial x \partial y}$ is :

(A) $\dfrac{\partial z}{\partial y} = \dfrac{y^2}{2} x^2 + f(x)$

(B) $\dfrac{\partial z}{\partial y} = 3x^3 y^2$

(C) $\dfrac{\partial z}{\partial y} = \dfrac{x^3}{3} y + f(y)$

(D) $\dfrac{\partial z}{\partial y} = 2xy + f(x)$

P.T.O.

(*iii*) The partial differential equation $z = (x + a)(y + b)$, on elimination of arbitrary constants, $\frac{\partial z}{\partial x}$ is :

(A) $(y + b)$ (B) $(x + a)$

(C) $(a + b)$ (D) xy

(*iv*) Solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is :

(A) $z = f_1(y + x) + f_1(y - x)$

(B) $z = f_2(y + x) + f_2(y - x)$

(C) $z = f(x^2 - y^2)$

(D) $z = f_1(y + x) + f_2(y - x)$

(*v*) The roots of the equation $(D^2 - DD' - D'^2)\, z = (y - 1)e^x$ are :

(A) (–2, 1) (B) (–2, –1)

(C) (2, –1) (D) (2, 1)

(*vi*) An equation :

$$a_0 \frac{\partial^n z}{\partial x^n} + a_1 \frac{\partial^n z}{\partial x^{n-1} \partial y} + \text{..........} + a_n \frac{\partial^n z}{\partial y^n} = f(x, y)$$

is called :

(A) Non-homogeneous (B) Homogeneous

(C) Auxiliary equation (D) None of these

(*vii*) In equation $f_1(x, p) = f_2(y, q)$, which of the following is *true* ?

(A) z is absent

(B) The term containing x and p

(C) The term containing y and q

(D) All of the above

(*viii*) Identify solution of the equation :

$$y^2 p - xyq = x(z - 2y)$$

(A) $x^2 + y^2 = f(y^2 - yz)$ (B) $f(x^2 + y^2, y^2 - yz) = 0$

(C) Both (A) and (B) (D) None of these

(*ix*) The PDE $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ is called :

(A) One-dimensional heat flow

(B) Wave equation

(C) Radio equations

(D) Two-dimensional heat flow

(*x*) Identify the Laplace equation in two-dimensions :

(A) $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ (B) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(C) $-\frac{\partial V}{\partial x} = L\frac{\partial I}{\partial t}$ (D) None of these

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) Explain the method of multipliers for finding solution of partial differential equation $Pp + Qq = R$ with usual notations.

(*b*) Form a partial differential equation from :

$$x^2 + y^2 + (z - c)^2 = a^2.$$

(*c*) Solve :

$$zp + yq = x.$$

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) Explain rules for finding the particular integral for given partial differential equation :

$$f(D, D')\, Z = F(x, y)$$

(*i*) when $F(x, y) = e^{ax+by}$.

(*ii*) when $F(x, y) = \sin(ax + by)$.

(*b*) Solve $p(1 + q^2) = q(z - a)$.

(*c*) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$.

4. Attempt any *two* of the following : 5 each

(*a*) Use the method of separation of variables to solve the equation :

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t}.$$

(*b*) Find the solution of $\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial u}{\partial t}$, for which $u(0, t) = u(l, t) = 0$, $u(x, 0) = \sin \frac{\pi x}{l}$ by method of variables separable.

(*c*) Solve :

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{d\theta^2} = 0$$

by the method of separation of variables.