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B—82—2019

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MARCH/APRIL, 2019

(CBCS/CGPA Pattern)

MATHEMATICS

Paper IX

(Real Analysis—II)

(MCQ & Theory)

(Wednesday, 27-3-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes are for Q. No. **1** (MCQ) and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken circle of correct choice in OMR answer-sheet.

(*v*) Negative marking system is applicable for MCQ.

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) The norm of a partition P is denoted as $\mu(P) =$

(*a*) $|x_i - x_{i-1}|$ (*b*) $x_i - x_{i-1}$

(*c*) $\max\limits_{1 \le i \le n} |x_i - x_{i-1}|$ (*d*) 1

(*ii*) If P^* is a common refinement of partitions P_1 and P_2, then :

(*a*) $P^* = P_1 \cap P_2$ (*b*) $P^* = P_1 \cup P_2$

(*c*) $P^* = P_1$ (*d*) $P^* = P_2$

(*iii*) If P^* is a refinement of P of $[a, b]$, then for a bounded function f,

(*a*) $L(P^*, f) \ge L(P, f)$ (*b*) $L(P, f) \ge L(P^*, f)$

(*c*) Both (*a*) and (*b*) (*d*) Neither (*a*) nor (*b*)

P.T.O.

(*iv*) The Riemann sum of f over $[a, b]$ relative to partition P is given by :

(*a*) $S(P, f) = \sum_{i=1}^{n} M_i \Delta x_i$

(*b*) $S(P, f) = \sum_{i=1}^{n} m_i \Delta x_i$

(*c*) $S(P, f) = \sum_{i=1}^{n} f(t_i) \Delta x_i$

(*d*) None of these

(*v*) A derivable function F, if it exists such that its derivative $F' = f$ is called :

(*a*) Norm of f

(*b*) Primitive of f

(*c*) Upper integral of f

(*d*) Lower integral of f

(*vi*) Functions possessing primitives are :

(*a*) Necessarily continuous

(*b*) Not necessarily continuous

(*c*) Necessarily derivable

(*d*) None of these

(*vii*) Which of the following is an improper integral ?

(*a*) $\int_{1}^{\infty} \frac{dx}{x^2}$

(*b*) $\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$

(*c*) $\int_{-1}^{\infty} \frac{dx}{x^2}$

(*d*) All of these

(*viii*) A series of the form $\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ is called :

(*a*) Fourier series

(*b*) Power series

(*c*) Trigonometric series

(*d*) Series

(*ix*) If the function is .... on the interval $[-\pi, \pi]$, then also its Fourier coefficients approach zero as $n \to \infty$.

(*a*) Continuous

(*b*) Piecewise continuous

(*c*) Not continuous

(*d*) Derivable

(*x*) If f is an odd function $f(-x) = -f(x)\ \forall\ x$, then $f \cos nx$ is an odd function is :

(*a*) $a_n = \int_{-\pi}^{\pi} f \cos nx\, dx$ (*b*) $a_n = -\frac{1}{\pi}\int_{-\pi}^{\pi} f \cos nx\, dx$

(*c*) $a_n = \int_{\pi}^{-\pi} f \cos nx\, dx$ (*d*) $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f \cos nx\, dx$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If f is a bounded function on $[a, b]$, then prove that to every $\in > 0$, there corresponds $\delta > 0$ such that :

$$U(P, f) < \int_a^b f\, dx + \in .$$

(*b*) Prove that every continuous function is integrable.

(*c*) Show that :

$$\int_1^2 f\, dx = \frac{11}{2},$$

where $f(x) = 3x + 1$.

3. Attempt any *two* of the following : 5 each

(*a*) If a function f is bounded and integrable on $[a, b]$, then the function F defined as :

$$F(x) = \int_a^x f(t)\, dt,\ a \le x \le b,$$

is continuous on $[a, b]$ and furthermore, if f is continuous at a point c of $[a, b]$, then prove that F is derivable at c and $F'(c) = f(c)$.

P.T.O.

(*b*) If f and g be two positive functions in $[a, b]$ such that :

$$\lim_{x \to a^+} \frac{f(x)}{g(x)} = l,$$

where, l is a non-zero finite number, then prove that the two integrals :

$$\int_a^b f\,dx \text{ and } \int_a^b g\,dx$$

converge and diverge together at a.

(*c*) Test the convergence of

$$\int_0^{\pi/2} \frac{\sin x}{x^p}\,dx.$$

4. Attempt any *two* of the following : 5 each

(*a*) For a periodic function of period 2π, prove that :

(*i*) $\int_{\alpha}^{\beta} f\,dx = \int_{\alpha+2\pi}^{\beta+2\pi} f\,dx$

(*ii*) $\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} f(\gamma + x)dx$

where α, β, γ being any numbers.

(*b*) If f is bounded and integrable on $[-\pi, \pi]$ and if a_n, b_n are its Fourier coefficients, then prove that :

$$\sum_{n=1}^{\infty} \left(a_n^2 + b_n^2\right)$$

converges.

(*c*) Find the Fourier series generated by the periodic function $|x|$ of period 2π. Also compute the value of series at $x = 0$.