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Y—100—2019

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) (Backlog) EXAMINATION

OCTOBER/NOVEMBER, 2019

MATHEMATICS

Paper (X)

(Ring Theory)

(MCQ+Theory)

(Tuesday, 19-11-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes are for Q. No. **1** (MCQ) and remaining time for other questions.

(*iii*) Use black ball pen to darken circle of correct choice in OMR answer sheet.

(*iv*) Figures to the right indicate full marks.

(*v*) Negative marking scheme is applicable for MCQ.

(MCQ)

1. Choose the correct alternative for each of the following : 1 each

(*i*) R is the set of integers, positive, negative and O; '+' is usual addition and '.' is usual multiplication of integers, then :

(*a*) R is a commutative ring

(*b*) R is a commutative ring with unit element

(*c*) R is a commutative ring but has no unit element

(*d*) R is not a ring

(*ii*) A ring is said to be a division ring if its :

(*a*) Nonzero elements form a group under addition

(*b*) Nonzero elements form a group under multiplication

(*c*) Zero elements form a group under addition

(*d*) Zero elements form a group under multiplication

P.T.O.

(*iii*) If n objects are distributed over m places and if $n > m$, then :

(*a*) All places receive exactly one object

(*b*) All places receive atmost one object

(*c*) Some places receive at least two objects

(*d*) Some places receive no object

(*iv*) Which of the following is/are true ?

(*a*) If $a|b$ and $b|c$ then $a|c$

(*b*) If $a|b$ and $a|c$ then $a|(b \pm c)$

(*c*) If $a|b$ and $a|bx$ for all $x \in$ R

(*d*) All of the above

(*v*) A ring R is said to be imbedded in a ring R′, if there exists :

(*a*) f is a homomorophism (*b*) f is a one-to-one

(*c*) both (*a*) and (*b*) (*d*) None of these

(*vi*) An ideal $M \neq R$ in a ring R is said to be maximal ideal of R if whenever U is an ideal of R such that $M \subset U \subset R$, thene :

(*a*) $R \neq U$ and $M \neq U$ (*b*) Either $R = U$ or $M = U$

(*c*) $R = U$ but $M \neq U$ (*d*) None of these

(*vii*) Let R be a commutative ring with unit element. Two elements a and b in R are said to be associates if :

(*a*) $b = ua$ for any u in R (*b*) $b = ua$ for some unit u in R

(*c*) $b \neq ua$ for u in R (*d*) $b \neq a$

(*viii*) If $p(x) = 1 + x + x^2$ and $q(x) = 2 - x^2 + x^3$, then $p(x) \cdot q(x) =$

(*a*) $2 + 2x + x^2 + x^5$ (*b*) $3 + x^2 - x^3 + x^5$

(*c*) $-7 + 2x + x^4$ (*d*) None of these

(*ix*) The polynomial $f(x) = a_0 + a_1 x + \ldots\ldots + \ldots + a_n x^n$, where $a_0, a_1, \ldots\ldots a_n$ are integers is said to be primitive if :

(*a*) The greatest common divisor of $a_0, a_1, a_2, \ldots\ldots a_n$ is greater than 1

(*b*) The least common multiple of $a_0, a_1, a_2, \ldots\ldots, a_n$ is greater than 1

(*c*) The greatest common divisor of $a_0, a_1, a_2, \ldots\ldots a_n$ is 1

(*d*) The least common multiple of $a_0, a_1, \ldots\ldots a_n$ is 1 (*x*) A polynomial is said to be integer monic if all its coefficients are :

(*a*) Integers and its highest coefficient is 1

(*b*) Natural and its highest coefficient is 1

(*c*) Integers and its highest coefficient is more than 1

(*d*) Integers and its highest coefficient is less than 1

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) Prove that a finite integral domain is a field.

(*b*) If R is a ring, then for all $a, b \in R$ prove that :

(*i*) $a0 = 0a = 0$

(*ii*) $a(-b) = (-a)b = -(ab)$

(*c*) Let R be aring, $R' = R$ and define $\phi(x) = x$ for every $x \in R$, then show thatn ϕ is a homomorphism and hence find kernel of ϕ.

3. Attempt any *two* of the folloiwng : 5 each

(*a*) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is field.

(*b*) Prove that the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R.

(*c*) Prove that if $[a, b] = [a', b']$ and $[c, d] = [c', d']$ then, $[a, b]\ [c, d] = [a', b']\ [c', d']$

4. Attempt any *two* of the following : 5 each

(*a*) If p is a prime number of the form $4n + 1$, then prove that $p = a^2 + b^2$ for some integers a, b.

(*b*) Prove that F[*x*] is a principal ideal ring.

(*c*) Show that $x^2 + x + 1$ is irreducible over F, where F is the field of integer mod 2.