This question paper contains **2** printed pages]

BF—49—2016

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

MATHEMATICS

Paper XIII (MT-301)

(Metric Spaces)

(Saturday, 15-10-2016) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define bounded metric space.

(*b*) Define open sphere of metric space.

(*c*) State Banach fixed point theorem.

(*d*) Write the conditions for a function f to be a homeomorphism in metric spaces.

(*e*) Define separated sets on a metric space.

(*f*) Define compact metric space.

2. Attempt any *two* of the following : 5 each

(*a*) In any metric space (X, d), prove that the union of an abitrary family of open sets is open.

(*b*) If A and B are two subsets of a metric space (X, d), then prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

P.T.O.

(*c*) Let X be the set of all sequences of complex numbers. We define the function :

$$d(x, y) = \sum_{n=1}^{\infty} \frac{1}{2^n} \frac{|x_n - y_n|}{(1 + |x_n - y_n|)}$$

for every $x = \{x_n\}$, $y = \{y_n\} \in$ X. Show that (X, d) is a metric space.

3. Attempt any *two* of the following : 5 each

(*a*) Let (X, d_1) and (Y, d_2) be two metric spaces, then prove that $f : X \to Y$ is continuous if and only if $f^{-1}(G)$ is open in X, whenever G is open in Y.

(*b*) If $f(x) = x^2$, $0 \le x \le \frac{1}{3}$, then prove that f is a contraction mapping on $\left[0, \frac{1}{3}\right]$ with the usual metric 'd'.

(*c*) Let (X, d_1) and (Y, d_2) be a metric spaces. Show that $f : X \to Y$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$, for every $A \subseteq X$.

4. Attempt any *two* of the following : 5 each

(*a*) Prove that every closed subset of compact metric space is compact.

(*b*) Prove that continuous image of a connected set is connected.

(*c*) Discuss the connectedness of the following subsets of the Euclidean space R^2 for the set :

$$D = \{(x, y) : x \neq 0 \text{ and } y = \sin 1/x\}.$$