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BF—60—2016

FACULTY OF SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

MATHEMATICS

Paper XIV (MT-302)

(Linear Algebra)

(Tuesday, 18-10-2016) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :—(*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define the internal direct sum.

(*b*) Define the subspace of a vector space V.

(*c*) For $\alpha \in F$ and $u \in V$ prove that :

$$\|\alpha u\| = |\alpha| \, \|u\|$$

(*d*) Define an algebraic number.

(*e*) Define invertible mapping.

(*f*) Define rank of T.

2. Attempt any *two* of the following : 5 each

(*a*) If V is the internal direct sum of $u_1, u_2, ... u_n$, then prove that V is isomorphic to the external direct sum of $u_1, u_2, ... u_n$.

P.T.O.

(*b*) If A and B are finite-dimensional subspaces of a vector space V, then prove that A + B is finite-dimensional and :

$$\dim(A + B) = \dim(A) + \dim(B) - \dim(A \cap B)$$

(*c*) In a vector space show that :

$$\alpha(v - w) = \alpha v - \alpha w.$$

3. Attempt any *two* of the following : 5 each

(*a*) If $u, v \in V$ and $\alpha, \beta \in F$, then prove that :

$$(\alpha u + \beta v, \alpha u + \beta v) = \alpha\bar{\alpha}(u, u) + \alpha\bar{\beta}(u, v) + \bar{\alpha}\beta(v, u) + \beta\bar{\beta}(v, v)$$

(*b*) Let V be a finite-dimensional inner product space, then prove that V has an orthonormal set as a basis.

(*c*) In vector space V, define the distance $d(u, v)$ from u to v by

$$d(u, v) = \|u - v\|$$

then prove that :

(*i*) $d(u, v) \geq 0$ and $d(u, v) = 0$ if and only if $u = v$

(*ii*) $d(u, v) = d(v, u)$

(*iii*) $d(u, v) \leq d(u, w) + d(w, v)$

4. Attempt any *two* of the following : 5 each

(*a*) If A is an algebra, with unit element, over F, then prove that A is isomorphic to a subalgebra of A(V) for some vector space V over F.

(*b*) If V is finite-dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

(*c*) Let V be two-dimensional over the field F, of real numbers, with a basis V_1, V_2. Find the characteristic roots and corresponding characteristic vectors for T defined by

$$V_1T = V_1 + V_2,$$

$$V_2T = V_1 - V_2.$$