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BF—74/75—2016

FACULTY OF SCIENCE/ARTS

B.Sc./B.A. (Third Year) (Fifth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

MATHEMATICS

Paper XV (A)

(Operation Research)

Or

Paper XV (B)

[Mechanics—I (Statics)]

(Thursday, 20-10-2016) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

Paper XV (A)

(Operation Research)

N.B. :—(*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) What is linear programming.

(*b*) Define alternative optima.

(*c*) Define associated cost vector.

(*d*) State *two* methods for solving linear programming problems having artificial variables.

(*e*) Define prohibited assignments.

(*f*) What is assignment problem ?

2. Attempt any *two* of the following : 5 each

(*a*) Explain the standard form of linear programming problem.

(*b*) Write the mathematical formulation of the following general linear programming problem :

Given nutrient contents of a number of different foodstuffs and the daily .... minimum requirement of each nutrient for a diet, determine the balanced diet which satisfied the minimum daily requirements and at the same time has minimum cost.

(*c*) Solve the following linear programming problem using Graphical method :

Maximize $z = 4x_1 + 3x_2$

Subjected to the contstraints

$$2x_1 + x_2 \leq 1000$$
$$x_1 + x_2 \leq 800$$
$$x_1 \leq 400 \text{ and } x_2 \leq 700$$
$$x_1 \geq 0 \text{ and } x_2 \geq 0.$$

3. Attempt any *two* of the following : 5 each

(*a*) Let X_B be a basic feasible solution to the L.P.P.

$$\text{Max } Z = CX,$$
$$\text{subject to } AX = b,$$
$$X \geq 0.$$

Let $\hat{X}_B$ be another basic feasible solution obtained by admitting a non-basis column vector a_j in the basis, for which net evaluation $z_j - c_j$ is negative. Then prove that $\hat{X}_B$ is an improved basic feasible solution to the problem :

i.e. $\hat{C}_B \hat{X}_B > C_B X_B$.

(*b*) Prove that a sufficient condition for a basic feasible solution to an L.P.P. to be an optimum is that $z_j - c_j \geq 0$ for all j, for which the .... column vector $a_j \in$ A is not in basis B.

(*c*) Let

$$x_1 = 2,\ x_2 = 4 \text{ and } x_3 = 1$$

be a feasible solution to the system of equations :

$$2x_1 - x_2 + 2x_3 = 2$$

$$x_1 + 4x_2 = 18$$

Reduce the given feasible solution to a basic feasible solution.

4. Attempt any *two* of the following : 5 each

(*a*) In an assignment problem, if we add or substract a constant to every element of any row or column of the cost matrix $[c_{ij}]$ then prove that an assignment that minimizes the total cost on one matrix also minimizes the total cost on other matrix. In other words, if $x_{ij} = x_{ij}^*$ minimizes

$$z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\ x_{ij},\ \sum_{i=1}^{n} x_{ij} = 1,\ \sum_{j=1}^{n} x_{ij} = 1$$

$x_{ij} = 0$ or 1. then x_{ij}^* also minimizes

$$z^* = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^* x_{ij} \text{ where } = c_{ij}^* = c_{ij} - u_i - v_j$$

for all i, j = 1, 2, ...n and u_i, v_j are some real numbers.

(*b*) The following is the cost matrix of assigning 4 clerks to 4 key punching Jobs. Find the optimal assignment if clerk 1 cannot be assignment to Job 1.

Job

Clerk	**1**	**2**	**3**	**4**
1	–	5	2	0
2	4	7	5	6
3	5	8	4	3
4	3	6	6	2

What is the minimum total cost ?

(*c*) A machine tool company decides to make four subassemblies through four contractors. Each contractor is to receive only one subassembly. The cost of each subassembly is determined by the bids submitted by each contractor and shown in the following table in hundreds of rupees.

Contractors

		1	**2**	**3**	**4**
	1	15	13	14	17
	2	11	12	15	13
Subassemblies	**3**	13	12	10	11
	4	15	17	14	16

Assign the different subassemblies to contractors so as to minimize the total cost.

OR

Paper XV (B)

[Mechanics—I (Statics)]

N.B. :—(*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define unlike parallel forces.

(*b*) State the law of the parallelogram of forces.

(*c*) If two forces $\vec{P}$ and $\vec{Q}$ act along the same straight line and in the same direction. Then find magnitude and direction of the resultant of two forces $\vec{P}$ and $\vec{Q}$.

(*d*) State the Lami's theorem.

(*e*) Find the vector moment of a force

$$\vec{F} = \vec{i} + 2\vec{j} + 3\vec{k}$$

acting at a point (–1, 2, 3) about the origin.

(*f*) Define vector moment of the force $\vec{F}$ about O.

2. Attempt any *two* of the following : 5 each

(*a*) Find the magnitude and direction of the resultant of any number of coplanar forces acting at a point.

(*b*) Prove that the resultant of two forces given by $m.\overrightarrow{OA}$ and $n.\overrightarrow{OB}$ is represented by $(m + n)\overrightarrow{OC}$, where the point C divides AB internally in the ratio $n : m$.

(*c*) Two forces $\overrightarrow{P}$ and $\overrightarrow{Q}$ act at a point along two lines making an angle θ with each other and $\overrightarrow{R}$ is their resultant. The magnitude of the resolved part of R in the direction of the force $\overrightarrow{P}$ is Q. Prove that :

$$\sin\frac{\theta}{2} = \sqrt{\frac{P}{2Q}}.$$

3. Attempt any *two* of the following : 5 each

(*a*) State and prove Triangle Law of forces.

(*b*) Prove that necessary and sufficient condition for a system of forces acting on a particle to be in equilibrium is that the algebraic sum of the resolved parts of the given forces along any three non-coplanar directions must separately vanish.

(*c*) A particle is placed at the centre O of the circle inscribed in a Δ ABC. Forces $\overrightarrow{P}, \overrightarrow{Q}, \overrightarrow{R}$ acting along $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively are in equilibrium prove that :

$$P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}.$$

4. Attempt any *two* of the following : 5 each

(*a*) Prove that magnitude of the moment of the couple equals to the product of magnitude of a force in the couple and arm of the couple.

(*b*) Find the conditions of equilibrium of forces acting on a rigid body in Cartesian co-ordinates.

(*c*) Three forces $\vec{P}, \vec{Q}, \vec{R}$ act along the sides BC, CA, AB of a Δ ABC, taken in order; prove that if the resultant passes through the incentre of Δ ABC, then $P + Q + R = 0$ where P, Q, R are the magnitudes of the forces.