This question paper contains **2** printed pages]

R—47—2017

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

MARCH/APRIL, 2017

MATHEMATICS

Paper XIII (MT-301)

(Metric Spaces)

(Wednesday, 29-3-2017) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define closed set in a metric space (X, *d*).

(*b*) Define subspace of a metric space (X, *d*).

(*c*) Define complete metric space.

(*d*) Define contraction mapping.

(*e*) State Heine-Borel Theorem.

(*f*) Define connected set of a metric space.

2. Attempt any *two* of the following : 5 each

(*a*) In any metric space (X, *d*), prove that the intersection of a finite number of open sets is open.

(*b*) Let A and B be any *two* subsets of metric space (X, *d*), then prove that $A = \bar{A}$ if and only if A is closed.

(*c*) Prove that every open sphere is a neighbourhood of each of its points.

3. Attempt any *two* of the following : 5 each

(*a*) Prove that every convergent sequence is a Cauchy sequence.

P.T.O.

(*b*) Let (X, d_1) and (Y, d_2) be two metric spaces, then prove that $f : X \to Y$ is continuous if and only if $f^{-1}(G)$ is open in X, whenever G is open in Y.

(*c*) Let (X, d_1) and (Y, d_2) be metric spaces, show that $f : X \to Y$ is continuous if and only if $f(\bar{A}) \subseteq \overline{f(A)}$ for every $A \subseteq X$.

4. Attempt any *two* of the following : 5 each

(*a*) Prove that, every compact subset A of a metric space (X, *d*) is bounded.

(*b*) Let A be a connected subset of a metric space X, and let B be a subset of X such that $A \subseteq B \subseteq \bar{A}$, then prove that B is also connected.

(*c*) Let A be a non-empty compact subset of a metric space (X, *d*) and let F be a closed subset of X such that $A \cap F = \varphi$, then prove that $d(A, F) > 0$.