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R—63—2017

FACULTY OF SCIENCE/ARTS

B.Sc./B.A. (Third Year) (Fifth Semester) EXAMINATION

MARCH/APRIL, 2017

MATHEMATICS

Paper XIV

(Linear Algebra)

(Friday, 31-3-2017) **Time : 10.00 a.m. to 12.00 noon**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 marks each

(*a*) Define vector space over a field F.

(*b*) Define linear combination over a field F.

(*c*) Define an element is algebraic over a field F.

(*d*) Define an element is of 'algebraic of degree n' over a field F.

(*e*) Define eigenvalue of linear transformation T.

(*f*) Define right invertible mapping.

2. Attempt any *two* of the following : 5 each

(*a*) Prove that the linear span L(*s*) is a subspace of vector space V.

(*b*) Prove that, if $v_1, v_2, \ldots\ldots\ldots, v_n$ are in vector space V, then either they are linearly independent or some v_k is a linear combination of the preceding ones, $v_1, v_2, \ldots\ldots\ldots, v_{k-1}$.

(*c*) Find the rank of the system of homogeneous linear equations overfield F, the field of real numbers, and find all the solutions :

$$x_1 + 2x_2 - 3x_3 + 4x_4 = 0$$

$$x_1 + 3x_2 - x_3 = 0$$

$$6x_1 + x_3 + 2x_4 = 0 .$$

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) Prove that orthogonal complement $W^{\perp}$ is a subspace of vector space V.

(*b*) If the vector space V is a finite-dimensional inner product space and W is a subspace of V, then prove that $(W^{\perp})^{1} = W$.

(*c*) If W is a subspace of vector space V and $v \in V$ satisfies $(v, w) + (w, v) \leq (w, w)$ for every $w \in W$, then prove that $(v, w) = 0$ for every $w \in W$.

4. Attempt any *two* of the following : 5 each

(*a*) If V is finite-dimensional over a field F, then prove that $T \in A(V)$ is singular if and only if there exists $av \neq 0$ in V such that vT = 0.

(*b*) Prove that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $v \neq 0$ in V, $vT = \lambda v$.

(*c*) Compute the matrix products :

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 1/3 & 1/3 \end{pmatrix}^2 .$$