This question paper contains **2** printed pages]

V—46—2017

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2017

MATHEMATICS

Paper XIII (MT-301)

(Metric Spaces)

(Friday, 13-10-2017) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define diameter of a Non-empty set in a metric space.

(*b*) Define adherent point in a metric space.

(*c*) Define Cauchy sequence.

(*d*) Define isometry function in a metric space.

(*e*) Define open cover.

(*f*) Define relatively compact set.

2. Attempt any *two* of the following : 5 each

(*a*) Prove that in any metric space (X, d) the intersection of an arbitrary family of closed set is closed.

(*b*) Let A and B be any two subsets of a metric space (X, d) then prove that :

$$\left(\overline{A \cup B}\right) = \bar{A} \cup \bar{B}.$$

P.T.O.

(*c*) Let (X, d) be any metric space. Show that the function d_1 defined by :

$$d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \ \forall \ x, y \in X$$

is a metric on X.

3. Attempt any *two* of the following : 5 each

(*a*) Let (X, d) be a complete metric space and Y be a subspace of X, then prove that Y is complete if and only if it is closed in (X, d).

(*b*) Prove that the image of a Cauchy sequence under a uniformly continuous function is again a Cauchy sequence.

(*c*) Prove that the space C[0, 1] of all bounded continuous real-valued functions defined on the closed interval [0, 1] with metric d given by :

$$d(f, g) = \max_{0 \le x \le 1} |f(x) - g(x)|$$

is a complete metric space.

4. Attempt any *two* of the following : 5 each

(*a*) Prove that every compact subset F of a metric space (X, d) is closed.

(*b*) Let Y be a subset of a metric space (X, d) and if Y is connected then prove that Y cannot be expressed as disjoint union of two non-empty closed sets in Y.

(*c*) Prove that a subset A of a metric space (X, d) is totally bounded it and only it $\bar{A}$ is totally bounded.