This question paper contains **2** printed pages]

# V—61—2017

## FACULTY OF ARTS/SCIENCE

## B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

## OCTOBER/NOVEMBER, 2017

MATHEMATICS

Paper XIV (MT-302)

(Linear Algebra)

**(Friday, 10-11-2017)** **Time : 10.00 a.m. to 12.00 noon**

*Time*—2 *Hours* *Maximum Marks*—40

*N.B.* :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2×5=10

(*a*) Define Homomorphism.

(*b*) Define linearly independent.

(*c*) Define Norm of V.

(*d*) Define orthogonal complement of W.

(*e*) If $T \in A(V)$, then define range of T.

(*f*) Define characteristic vector of T.

2. Attempt any *two* of the following : 5×2=10

(*a*) If $v_1, v_2 \ldots\ldots\ldots v_n \in V$ are linearly independent, then prove that every element in their linear span has a unique representation in the form $\lambda_1 v_1 + \lambda_2 v_2 + \ldots\ldots\ldots + \lambda_n v_n$ with $\lambda_i \in F$.

(*b*) If $v$ is finite-dimensional and $w$ is a subspace of $v$, then prove that $A(A(w)) = w$.

(*c*) If F is the field of real numbers, then prove that the vectors (1, 1, 0, 0), (0, 1, –1, 0) and (0, 0, 0, 3) in $F^{(4)}$ are linearly independent over F.

P.T.O.

3. Attempt any *two* of the following : 5×2=10

(*a*) If $u, v \in V$, then prove that :

$$|(u, v)| \leq \|u\| \, \|v\|.$$

(*b*) If V is a finite-dimensional inner product space and W is a subspace of V, then prove that $(W^{\perp})^{\perp} = W$.

(*c*) In *v*, prove the parallelogram law :

$$\|u + v\|^2 + \|u - v\|^2 = 2\,(\|u\|^2 + \|v\|^2)$$

4. Attempt any *two* of the following : 5×2=10

(*a*) If A be an algebra, with unit element, over F, and suppose that A is of dimension *m* over F, then prove that every element in A satisfies some non-trivial polynomial in F[*x*] of degree at mast *m*.

(*b*) If V is finite-dimensional over F then for S, T $\in$ A(A), prove that :

(*i*) $r(ST) \leq r(T)$

(*ii*) $r(TS) \leq r(T)$

(*c*) Let V be two-dimensional over the field F, of real numbers, with a basis $v_1$, $v_2$. Find the characteristic root and corresponding characteristic vectors for T defined by

$$V_1T = 5V_1 + 6V_2, \; V_2T = -\,TV_2.$$