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V—72—2017

FACULTY OF SCIENCE/ARTS

B.Sc./B.A. (Third Year) (Fifth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2017

MATHEMATICS

Paper—XV (303-A)

(Operation Research)

(Monday, 13-11-2017) **Time : 10.00 a.m. to 12.00 noon**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) What are the components of linear programming model ?

(*b*) Define Optimum solution.

(*c*) Define Basic feasible solution.

(*d*) define a basic solution to a given system of m simultanesous linear equation in a n unknowns.

(*e*) Give the mathematical formulation of an assignment problem.

(*f*) How many solutions for n workers and n jobs ?

2. Attempt any *two* of the following : 5 each

(*a*) Explain the canonical form of L.P.P.

(*b*) Solve the following linear programming problem by Graphical method.

Maximize $z = 0.10x_1 + 0.20x_2$

Subject to the constraints :

$$x_1 + x_2 \leq 1,00,000$$

$$-0.02x_1 + 0.08x_2 \geq 0$$

P.T.O.

$$-2x_1 + 3x_2 \le 0$$
$$x_1 \le 75{,}000$$
$$x_2 \le 75{,}000$$
$$x_1 \ge 0 \text{ and}$$
$$x_2 \ge 0.$$

(*c*) Rewrite in standard form the following linear programming problem :

Minimize $z = 2x_1 + x_2 + 4x_3$

Subject to the constraints :

$$-2x_1 + 4x_2 \le 4,$$
$$x_1 + 2x_2 + x_3 \ge 5,$$
$$2x_1 + 3x_3 \le 2.$$
$$x_1, x_2 \ge 0 \text{ and}$$

x_3 unrestricted in sign.

3. Attempt any *two* of the following : 5 each

(*a*) If the feasible region of an L.P.P. is a convex polyhedron, then prove that there exist an optimal solution to the L.P.P. and at least one basic feasible solution must be optimal.

(*b*) Prove that any convex combination of *k* different optimum solution to an L.P.P. is again an optimum solution to the problem.

(*c*) Show that the following system of linear equations has a degenerate solution.

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3$$

4. Attempt any *two* of the following : 5 each

(*a*) Explain the Hungarian Assignment method.

(*b*) A company wishes to assign 3 job to 3 machines in such a way that each job is assinged to some machine and no machine works on more than one job. The cost of assigning job i to machine j is given by the matrix below :

$$\text{Cost matrix : } \begin{bmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{bmatrix}$$

Draw the associated network and formulate the network L.P.P.

(*c*) A student has to select one and only one elective in each semester and the same elective should not be selected in different semesters. Due to various reasons, the expected grades in each subject, if selected in different semesters, vary and they are given below :

Semester	**Analysis**	**Statistics**	**Graph Theory**	**Algebra**
I	F	E	D	C
II	E	E	C	C
III	C	D	C	A
IV	B	A	H	H

The grade points are : H = 10, A = 9, B = 8, C = 7, D = 6, E = 5 and F = 4. How will the student select the electives in order to maximize the total expected points and what will be his maximum expected total points ?