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AO—45—2018

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

MARCH/APRIL, 2018

MATHEMATICS

Paper XIII (MAT-301)

(Metric Spaces)

(Thursday, 22-3-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt all questions.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define open set in a metric space (X, *d*).

(*b*) Define bounded metric space.

(*c*) Define Cauchy sequence.

(*d*) Define contraction mapping in metric space (X, *d*).

(*e*) State Heine-Borel theorem.

(*f*) Define connected metric space.

2. Attempt any *two* of the following : 5 each

(*a*) Let (X, *d*) be a metric space and Y $\subset$ X, then prove that a subset A of Y is open in (Y, *dy*) if and only if there exists a set G open in (X, *d*) such that :

$$A = G \cap y$$

(*b*) Let X be the set of all sequences of complex numbers, define the function *d* by :

$$d(x, y) = \sum_{n=1}^{\infty} \frac{1}{2^n} \frac{|x_n - y_n|}{(1 + |x_n - y_n|)}, \forall x = \{x_n\}, y = \{y_n\} \in X$$

Show that the function *d* is a metric on X i.e. (X, *d*) is a metric space.

P.T.O.

(*c*) Prove that every closed sphere is a closed set.

3. Attempt any *two* of the following : 5 each

(*a*) Let (X, d_1) and (Y, d_2) be two metric spaces, then prove that $f : X \to Y$ is continuous if and only if $f^{-1}(G)$ is open in X, whenever G is open in *y*.

(*b*) Prove that any contraction mapping *f* of *a* non-empty complete metric space (X, *d*) into itself has a unique fixed point.

For all $x, y \in X$,

we have $d(f(x), d(y)) \leq \alpha \, d(x, y)$.

(*c*) Let (X, d_1) and (Y, d_2) be metric spaces. Show that $f : X \to Y$ is continuous if and only if $f(\overline{A}) \subset \overline{f(A)}$, for every $A \subset X$.

4. Attempt any *two* of the following : 5 each

(*a*) Prove that every compact subset A of a metric space (X, *d*) is bounded.

(*b*) Prove that continuous image of a connected set is connected.

(*c*) Prove that a subset A of a metric space (X, *d*) is totally bounded if and only if $\overline{A}$ is totally bounded.