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AO—60—2018

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

MARCH/APRIL, 2018

MATHEMATICS

Paper XIV (MT-302)

(Linear Algebra)

(Saturday, 24-03-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define homomorphism mapping in a vector space.

(*b*) Define linear span in vector space.

(*c*) Define inner product space.

(*d*) If $u \in V$, $\alpha \in F$, then prove that :

$$||\alpha u|| = |\alpha| \; ||u||.$$

(*e*) Define the term characteristic root of T.

(*f*) Compute : $\begin{pmatrix} 1 & 6 \\ -6 & 1 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}$

2. Attempt any *two* of the following : 5 each

(*a*) If $V_1, V_2, ..., V_n \in V$ are linearly independent, then prove that every element in their linear span has a unique representation in the form $\lambda_1 U_1 + \lambda_2 V_2 + ... + \lambda_n V_n$ with $\lambda_i \in F$.

P.T.O.

(*b*) If A and B are finite-dimensional subspaces of a vector space V, then prove that A + B is finite-dimensional and

$$\dim (A + B) = \dim (A) + \dim (B) - \dim (A \cap B).$$

(*c*) Prove that the intersection of two subspaces of V is a subspace of V.

3. Attempt any *two* of the following : 5 each

(*a*) If a, b, c are real numbers such that $a > 0$ and $a\lambda^2 + 2b\lambda + c \geq 0$ for all real numbers λ, then prove that :

$$b^2 \leq ac.$$

(*b*) If $u, v \in V$, then prove that :

$$|(u, v)| \leq \|u\| \, \|v\|$$

(*c*) If W is a subspace of V and if $v \in V$ satisfies $(v, w) + (w, v) \leq (w, w)$ for every $w \in W$, then prove that :

$$(v, w) = 0 \text{ for every } w \in W.$$

4. Attempt any *two* of the following : 5 each

(*a*) If V is a finite dimensional over F then for S, T ∈ A(V), prove that :

(*i*) $r(ST) \leq r(T)$

(*ii*) $r(TS) \leq r(T)$

(*b*) Prove that, the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $V \neq 0$ in V,

$$VT = \lambda V.$$

(*c*) Let V be two-dimensional over the field F, of real numbers, with a basis v_1, v_2. Find the characteristic roots and the corresponding characteristic vectors for T defined by

$$v_1T = 5v_1 + 6v_2, \quad v_2T = -7v_2.$$