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W—49—2018

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS Pattern)

MATHEMATICS

Paper XII

(Metric Spaces)

(Saturday, 13-10-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *four* of the following : 2 each

(*a*) Define an adherent point in a metric space.

(*b*) If $G_n = \left(-\frac{1}{n}, \frac{1}{n}\right)$, $\forall\ n \in N$, is open in the usual metric space (R, *d*), then determine whether $\bigcap_{n=1}^{\infty} G_n$ is open.

(*c*) Define Cauchy sequence.

(*d*) Write the condition for a function *f* to be a homeomorphism in a metric space.

(*e*) Define compact subset of a metric space.

(*f*) Define finite intersection property.

2. Attempt any *two* of the following : 4 each

(*a*) In any metric space (X, *d*), prove that the union of an arbitrary family of open sets is open.

P.T.O.

(*b*) Let (X, *d*) be any metric space. Then prove that a subset F, of X, is closed if and only if its complement, in X, is open.

(*c*) Let (X, *d*) be any metric space. Then show that the function 'd'' defined by :

$$d'(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \ \forall \ x, y \in X$$

is also a metric on X.

3. Attempt any *one* of the following : 8 each

(*a*) (*i*) Prove that every convergent sequence is a Cauchy sequence.

(*ii*) Let (X, d_1) and (Y, d_2) be metric spaces. Then show that $f : X \to Y$ is continuous if and only if $f(\bar{A}) \subseteq \overline{f(A)}$, for every $A \subseteq X$.

(*b*) (*i*) Let (X, d_1) and (Y, d_2) be any two metric spaces and f is a function from X into Y, then show that f is continuous at $a \in X$ if and only if for every sequence $\{a_n\}$ converging to a, we have $\lim_{n \to \infty} f(a_n) = f(a)$.

(*ii*) For any non-empty subset A, of a metric space (X, *d*), prove that the function $f : X \to R$ given by $f(x) = d(x, A)$, for $x \in X$ is uniformly continuous.

4. Attempt any *two* of the following : 4 each

(*a*) Prove that every compact subset A, of a metric space (X, *d*), is bounded.

(*b*) Prove that continuous image of a connected set is connected.

(*c*) Let A be a non-empty compact subset of a metric space (X, *d*) and F be a closed subset of X, such that $A \cap F = \phi$, then prove that $d(A, F) > 0$.

5. Attempt any *one* of the following : 8 each

(*a*) Let (X, d) be a metric space and $Y \subseteq X$, then prove that a subset A, of Y, is open in (Y, d_Y) if and only if there exists a set G, open in (X, d), such that $A = G \cap Y$.

(*b*) Let (X, d_1) and (Y, d_2) be two metric spaces, then show that $f : X \to Y$ is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

(*c*) Let Y be a subset of a metric space (X, d), then prove that the following statements are equivalent :

(*i*) Y is connected

(*ii*) Y cannot be expressed as disjoint union of two non-empty closed sets in Y.