This question paper contains **2** printed pages]

W—50—2018

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CGPA Pattern)

MATHEMATICS

Paper XIII (MT-301)

(Metric Spaces)

(Saturday, 13-10-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define Neighbourhood of a point.

(*b*) Define Subspace of a metric space (X, d)

(*c*) Define complete metric space.

(*d*) State Banach fixed point theorem.

(*e*) Define Compact metric space.

(*f*) Define separated sets on a metric space.

2. Attempt any *two* of the following : 5 each

(*a*) Let (X, d) be any metric space. Prove that A subset F of X is closed if and only if its complement in X is open.

(*b*) Let A and B be any two subsets of a metric space (X, d). Then prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

(*c*) Show that the set R^n of all ordered n-tuples with the function d defined by :

$$d(x, y) = \sum_{i=1}^{n} \left(xi - yi^2\right)^{1/2},$$

for all $x = (x_1, x_2, \ldots x_n)$, $y = (y_1, y_2 \ldots y_n) \in R^n$ is a metric space.

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) Let (X,d) be any metric space and A be any non-empty subset of X, then prove that $x \in \bar{A}$ if and only if there exists a sequence $\{x_n\}$ in A such that $x_n \to x$, as $n \to \infty$.

(*b*) Let (X,d_1) and (Y,d_2) be any two be any two metric spaces and f is a function from X into Y. Then prove that f is continuous at $a \in X$ if and only if for every sequence $\{a_n\}$ converging to 'a' we have $\lim_{x \to \infty} f(a_n) = f(a)$.

(*c*) If $f(x) = x^2$, $a \leq x \leq \frac{1}{3}$, then prove that f is a contraction mapping on $\left[0, \frac{1}{3}\right]$ with the usual metric d.

4. Attempt any *two* of the following : 5 each

(*a*) Prove that every compact subset A of a metric space (X,d) is bounded.

(*b*) Let A be a connected subset of a metric X, and let B be a subset of X such that $A \subseteq B \subseteq \bar{A}$, then prove that B is also connected.

(*c*) Discuss the connectedness of the following subset of the Euclidean space R^2 :

$D = \{(x,y) : x \neq 0 \text{ and } y = \sin 1/x.\}$.