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W—68—2018

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS Pattern)

MATHEMATICS

Paper XIII

(Linear Algebra)

(Tuesday, 16-10-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *four* of the following : 8

(*a*) Define vector space over a field.

(*b*) For a subset S of a vector space V over a field F, define linear span L(S) of S.

(*c*) In inner product space over a field, define norm of a vector.

(*d*) Define degree of extension of a field.

(*e*) What is eigenvalue of a Linear Transformation ?

(*f*) For a finite-dimensional vector space V over a field F, and for any $T \in A(V)$, define matrix of T in a basis of V.

2. Attempt any *two* of the following : 8

(*a*) If V is a vector space over a field F and if $V_1, V_2, ..., V_n$ are in V, then prove that either they are linearly independent or some V_k is a linear combination of preceding ones $V_1, V_2, ..., V_{k-1}$.

(*b*) If W is a subspace of a vector space V, then prove that its annihilator A(W) is a subspace of $\hat{V}$.

P.T.O.

(*c*) If F is the field of real numbers, then show that the vectors $V_1 = (1, 1, 2)$, $V_2 = (0, 1, 2)$ and $V_3 = (1, 2, 4)$ are linearly dependent in vector space $V = F^{(3)}$.

3. Attempt any *one* of the following : 8

(*a*) (*i*) State and prove Schwarz inequality.

(*ii*) If V is a finite-dimensional inner product space and if W is a subspace of V, then prove that $V = W + W^{\perp}$, where the sum of W & $W^{\perp}$ is direct sum.

(*b*) (*i*) Define orthonormal set of vectors in an inner product space V over F and prove that if $\{V_1, V_2, ..., V_n\}$ is an orthonormal set in V, then for any $w \in V$,

$$u = w - (w, V_1)V_1 - (w, V_2)V_2 - .... - (w, V_n)V_n$$

is orthogonal to each $V_1, V_2, ..., V_n$.

(*ii*) State and prove parallelogram law for vectors in inner product space.

4. Attempt any *two* of the following : 8

(*a*) Prove that if V is finite-dimensional vector space over F, then $T \in A(V)$ is singular if, and only if, there exists $V \neq 0$ in V such that $VT = 0$.

(*b*) For a vector space V over a field F, if $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that λ is a root of minimal polynomial of T. Also prove that if V is finite-dimensional, then T has only a finite number of characteristic roots in F.

(*c*) Compute the following matrix product :

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}^2$$

5. Attempt any *one* of the following : 8

(*a*) If V is a vector space over a field F, then prove each of the following :

(*i*) $\alpha 0 = 0$ for $\alpha \in F$

(*ii*) $0V = 0$ for $v \in V$

(*iii*) $(-\alpha)V = -(\alpha V)$ for $\alpha \in F$, $v \in V$

(*iv*) If $V \neq 0$, then $\alpha V = 0$ implies $\alpha = 0$.

(*b*) Let F be a real field, V be the set of all polynomials, in a variable x, over F, of degree 2 or less and inner product in V be defined by :

$$\left(p(x), q(x)\right) = \int_{-1}^{1} p(x)\, q(x)\, dx.$$

Starting with the basis $V_1 = 1$, $V_2 = x$, $V_3 = x^2$, obtain an orthonormal basis of V.

(*c*) If V is n-dimensional vector space over a field F and if $T \in A(V)$ has a matrix $m_1(T)$ in the basis $V_1, V_2, ..., V_n$ and the matrix $m_2(T)$ in the basis $W_1, W_2, ..., W_n$ of V over F, then prove that there is an element $C \in F_n$ such that :

$$m_2(T) = C m_1(T) C^{-1}.$$