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W—69—2018

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CGPA Pattern)

MATHEMATICS

Paper XIV (MT-302)

(Linear Algebra)

(Tuesday, 16-10-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*i*) Define subspace of a vector space.

(*ii*) Define basis of a vector space.

(*iii*) Define norm of vector $v \in V$, V is a vector space.

(*iv*) If W is a subspace of a vector space V over a field F, define orthogonal component of W.

(*v*) Define algebraic extension of a field.

(*vi*) Define rank of a linear transformation.

2. Attempt any *two* of the following : 5 each

(*i*) If S is a non-empty subset of the vector space V, then prove that the linear span L(S) is a subspace of V.

(*ii*) If V is the internal direct sum of subspaces $U_1, U_2, \ldots, U_n$, then prove that V is isomorphic to the external direct sum of $U_1, U_2, \ldots, U_n$.

(*iii*) If S, T are subsets of vector space V, then prove that, $S \subset T$ implies $L(S) \subset L(T)$.

3. Attempt any *two* of the following : 5 each

(*i*) If W is a subspace of vector space V, then prove that orthogonal complement of W is a subspace of V.

(*ii*) If V is a finite-dimensional inner product space and W is a subspace of V, then porve that $(W^{\perp})^{\perp} = W$.

P.T.O.

(*iii*) If V is a vector space, then show that in V, the parallelogram law :

$$\|u + v\|^2 + \|u - v\|^2 = 2\,(\|u\|^2 + \|v\|^2).$$

4. Attempt any *two* of the following : 5 each

(*i*) Let A be an algebra, with unit element, over a field F, and suppose that A is of dimension m over F. Then prove that every element in A satisfies some non-trivial polynomial in F(x) of degree at most m.

(*ii*) If vector space V is finite-dimensional over field F, then prove that, $T \in A(V)$ is singular if and only if there exists a $v \neq 0$ in V such that $vT = 0$.

(*iii*) If V be two-dimensional over the field F, of real numbers, with a basis v_1, v_2, find the characteristic roots and corresponding characteristic vectors for T defined by :

$$v_1 T = v_1 + v_2, \; v_2 T = v_1 - v_2.$$