This question paper contains **3** printed pages]

W—80—2018

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS Course)

MATHEMATICS

Paper XIV

(Operation Research)

(Friday, 19-10-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *four* of the following : 2 each

(*a*) What is unbounded solution ?

(*b*) Define standard form.

(*c*) Define Improved Basic feasible solution.

(*d*) Define Net evaluation.

(*e*) Define Degenerate solution.

(*f*) Define non-negative artificial variable.

2. Attempt any *two* of the following : 4 each

(*a*) Explain the major steps in the solution of linear programming problem by graphical method.

(*b*) The set of feasible solutions to an L.P.P. is a convex set. Prove it.

(*c*) Use the graphical method to solve the following LPP :

Maximize : $Z = 2x_1 + 3x_2$;

Subject to the constraints :

$$x_1 + x_2 \le 30,\ x_1 - x_2 \ge 0,\ x_2 \ge 3,$$
$$0 \le x_1 \le 20 \text{ and } 0 \le x_2 \le 12.$$

P.T.O.

3. Attempt any *one* of the following : 8

(*a*) (*i*) Prove that any convex combination of K different optimum solutions to an L.P.P. is again an optimum solution to the problem.

(*ii*) Maximize :

$Z = 3x_1 + 2x_2$

Subject to the constraints :

$2x_1 + x_2 \leq 2,\ 3x_1 + 4x_2 \geq 12,\ x_1,\ x_2 \geq 0$

(*b*) (*i*) Let X_B be a basic feasible solution to the L.P.P. : Maximize $Z = CX$, subject to $AX = b$, $X \geq 0$. Let $\hat{X}_B$ be another basic feasible solution obtained by admitting a non-basis column vector a_j in the basis, for which the net evaluation $Z_j - C_j$ is negative. Then prove that $\hat{X}_B$ is an improved basic feasible solution to the problem, that is $\hat{C}_B\hat{X}_B > C_B X_B$.

(*ii*) Use two-phase Simplex method to solve :

Maximize

$$Z = 5x_1 + 3x_2$$

Subject to the constraints :

$$2x_1 + x_2 \leq 1,\ x_1 + 4x_2 \geq 6 \text{ and}$$

$$x_1,\ x_2 \geq 0.$$

4. Attempt any *two* of the following : 4 each

(*a*) Explain the LP formulation of the Transportation problem.

(*b*) Explain special cases in Assignment problems.

(*c*) The following is the cost matrix of assigning 4 clerks to 4 key punching jobs. Find the optimal assignment if clerk 1 cannot be assigned to job 1 :

Clerk	**Job**			
	1	**2**	**3**	**4**
1	—	5	2	0
2	4	7	5	6
3	5	8	4	3
4	3	6	6	2

What is the minimum total cost ?

5. Attempt any *one* of the following : 8

(*a*) A farm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents (call them X, Y and Z), it is necessary to buy two additional products, say, A and B. One unit of product A contains 36 units of X, 3 units of Y and 20 units of Z. One unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirement of X, Y and Z is 108 units, 36 units and 100 units respectively. Product A costs Rs. 20 per unit and product B Rs. 40 per unit.

Formulate the above as a linear programming problem to minimize the total cost, and solve the problem by using graphic method.

(*b*) Let an L.P.P have a basic feasible solution. If we drop one of the basis vectors and introduce a non-basis vector in the basis set, then prove that new solution obtained is also a basic feasible solution.

(*c*) Explain the Hungarian Assignment problem.