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W—82—2018

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS Pattern)

MATHEMATICS

Paper-XIV

(Complex Analysis)

(Friday, 19-10-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) *All* questions carry equal marks.

1. Attempt any *four* of the following (each of **2** marks) : 8

(*a*) Define modulus of complex number. Give geometrical representation of modulus of z where $z = x + iy$.

(*b*) Find argument of (–1, –1).

(*c*) Define Entire function. Give *one* example.

(*d*) If $f(z) = z^2$, then $f(re^{i\theta}) = u(r, \theta) + iv(r, \theta)$, find $u(r, \theta)$ and $v(r, \theta)$.

(*e*) Obtain $\log(-1 - \sqrt{3}i)$ in terms of $a + ib$.

(*f*) Show that :

$$\log(i^3) \neq 3 \log i.$$

2. Attempt any *two* out of the following (each of **4** marks) : 8

(*a*) If $z_1 = -3 + 2i$ and $z_2 = 1 + 4i$, then which point is closer to origin and why ?

(*b*) If $z = x + iy$ is any complex number, find its :

(1) Additive inverse

(2) Multiplicative inverse.

State the condition when its multiplicative inverse exists.

(*c*) Find all values of $(-8i)^{1/3}$.

P.T.O.

3. Attempt any *one* of the following (each of 8 marks) : 8

(*a*) (*i*) Show that if $f(z) = \frac{iz}{2}$ in the open disk $|z| < 1$ then $\lim_{z \to 1} f(z) = \frac{i}{2}$.

(*ii*) If $f'(z) = 0$ everywhere in a domain D, then $f(z)$ must be constant throughout D.

(*b*) (*i*) If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then its component functions u and v are harmonic in D.

(*ii*) Obtain in harmonic conjugate of a given harmonic function :

$$u(x, y) = y^3 - 3x^2 y.$$

4. Attempt any *two* out of the following (each of 4 marks) : 8

(*a*) Show that :

$$2 \sin z_1 \,.\, \cos z_2 = \sin (z_1 + z_2) + \sin(z_1 - z_2).$$

(*b*) Find the value of $z = x + iy$ such that :

$$e^z = 1 + i.$$

(*c*) Show that for n = 0, ± 1, ± 2, ... $\log e = 1 + 2n\pi i$.

5. Attempt any *one* of the following (each of 8 marks) : 8

(*a*) Explain the method to find the nth roots of non-zero complex number z_0, and hence find the square root of $2i$.

(*b*) Let the function $f(z) = u(r, \theta) + iv(r, \theta)$ be defined throughout some $\in$ neighbourhood of a non-zero point $z_0 = r_0 \exp(i\theta_0)$, and suppose that the first order partial derivatives of the function u and v with respect to r and θ exist everywhere in that neighbourhood. If those partial derivatives are continuous at (r_0, θ_0) and satisfy the polar form :

$$ru_r = v_\theta,\ u_\theta = -rv_r$$

of the Cauchy-Riemann equations at (r_0, θ_0).

(*c*) Prove that :

$$\log z = \text{In } r + i(\theta + 2n\pi)\ (n = 0, \pm 1, \pm 2, \ldots\ldots..)$$

where z is any non-zero complex number. For $z = -1 - \sqrt{3}i$ find log 3.