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B—56—2019

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

MARCH/APRIL, 2019

(CBCS Pattern)

MATHEMATICS

Paper-XII

(Metric Spaces)

(Monday, 25-3-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *four* of the following : 2 each

(*a*) Define diameter of non-empty subset of metric space.

(*b*) If

$$F_n = \left[\frac{1}{n}, 1\right], \ \forall\, n \in N$$

is closed in the usual metric space, then determine whether $\bigcup_{n=1}^{\infty} F_n$ is closed.

(*c*) Define complete metric space.

(*d*) Define contraction mapping in metric space (X, d).

(*e*) Define an open cover of subset A of metric space (X, d).

(*f*) Define separated sets in a metric space.

P.T.O.

2. Attempt any *two* out of the following : 4 each

(*a*) In any metric space (X, d), prove that the union of an arbitrary family of open sets is open.

(*b*) Let A be any subset of a metric space (X, d). Then prove that $A = \bar{A}$ if and only if A is closed.

(*c*) Let (X, d) be any metric space. Show that the function d_1 defined by,

$$d_1(x, y) = \frac{d(x, y)}{1+d(x, y)}, \quad \forall\, x, y \in X$$

is a metric on X.

3. Attempt any *one* of the following : 8 each

(*a*) (*i*) Prove that, every convergent sequence is a Cauchy sequence.

(*ii*) Prove that, the space C[0, 1] of all bounded continuous real-valued functions defined on the closed interval [0, 1] with the metric "d" given by :

$$d(f, g) = \max_{0 \le x \le 1} |f(x) - g(x)|$$

is a complete metric space.

(*b*) (*i*) Prove that, the image of a Cauchy sequence under a uniformly continuous function is again a Cauchy sequence.

(*ii*) For any non-empty subset A of a metric space (X, d), prove that the function $f : X \to R$ given by, $f(x) = d(x, A)$ for $x \in X$ is uniformly continuous.

4. Attempt any *two* out of the following : 4 each

(*a*) Prove that, every closed subset of a compact metric space is compact.

(*b*) Prove that, continuous image of a connected set is connected.

(*c*) Discuss the connectedness of the subset D, of Euclidean space R^2, where $D = \{(x, y) / x \neq 0 \text{ and } y = \sin\frac{1}{x}\}$.

5. Attempt any *one* of the following : 8 each

(*a*) Let (X, d) be a metric space and $Y \subseteq X$, then a subset A of Y is open in (Y, d_1) if and only if there exist a set G, open in (X, d), such that $A = G \cap Y$.

(*b*) Let (X, d_1) and (Y, d_2) be two metric spaces, then $f : X \to Y$ is continuous if and only if $f^{-1}(G)$ is open in X, whenever G is open in Y.

(*c*) Prove that, every compact subset F of a metric space (X, d) is closed.