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B—57—2019

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

MARCH/APRIL, 2019

(CGPA Pattern)

MATHEMATICS

Paper-XIII, MT-301

(Metric Spaces)

(Monday, 25-3-2019)

Time : 10.00 a.m. to 12.00 noon

Time—2 Hours

Maximum Marks—40

N.B. :— (i) Attempt All questions.

(ii) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

- (a) Define metric on set X.
- (b) Define open sphere on metric space.
- (c) State Cantor's intersection theorem.
- (d) Define continuous metric space.
- (e) Prove that, the open interval $]0,1[$ with usual metric is not compact.
- (f) Define finite intersection property.

2. Attempt any *two* of the following : 5 each

- (a) In any metric space (X, d) , prove that the union of an arbitrary family of open sets is open.
- (b) Let (X, d) be any metric space. Show that the function d_1 defined by :

$$d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \forall x, y \in X,$$

d_1 is a metric on X.

- (c) Prove that, every closed sphere is a closed set.

P.T.O.

3. Attempt any *two* of the following : 5 each

- (a) Prove that, every convergent sequence is a Cauchy sequence.
- (b) Prove that, the image of a Cauchy sequence under a uniform continuous function is again Cauchy sequence.
- (c) Let X be set of all continuous real valued functions defined on $[0, 1]$ and let :

$$d(x, y) = \int_0^1 |x(t) - y(t)| dt,$$

$$x, y \in X$$

Show that (X, d) is not complete.

4. Attempt any *two* of the following : 5 each

- (a) Prove that, every closed subset of a compact metric space is compact.
- (b) A subset Y of a metric space X is disconnected if and only if $Y \subseteq G_1 \cup G_2$. Where G_1 and G_2 are open sets in X such that $Y \cap G_1 \neq \phi$, $Y \cap G_2 \neq \phi$, $G_1 \cap G_2 \cap Y = \phi$.
- (c) Discuss the connectedness of the subset :

$$D = \{(x, y) : x \neq 0 \text{ and } Y = \sin 1/x\}$$

of the Euclidean space \mathbb{R}^2 .