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B—79—2019

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

MARCH/APRIL, 2019

(CBCS)

MATHEMATICS

Paper-XIII

(Linear Algebra)

(Wednesday, 27-3-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *four of* the following : 8

(*a*) Define subspace of a vector space.

(*b*) Write a standard basis of $F^{(n)}$.

(*c*) State Schwarz inequality.

(*d*) Define algebraic element of an extension of a field.

(*e*) Define the term characteristic vector of T.

(*f*) Give examples of linear transformations S and T such that TS = 1 but ST ≠ 1.

2. Attempt any *two* of the following : 8

(*a*) If $V_1, V_2, \ldots\ldots\ldots, V_n$ is a basis of a vector space V over F and if $w_1, w_2, \ldots\ldots\ldots, w_m$ in V are linearly independent over F, then prove that $m \leq n$.

(*b*) If U and V are vector spaces over F and T : U → V is a homomorphism then prove that Kernel of T is a subspace of U.

(*c*) If W is a subspace of V, then prove that :

$$A(A(W)) = W.$$

P.T.O.

3. Attempt any *one* of the following : 8

(*a*) (*i*) If $\{V_i\}$ is an orthonormal set in an inner product space V then prove that the vectors $\{V_i\}$ are linearly independent. Also if $w = \alpha_1 V_1 + \alpha_2 V_2 + ... + \alpha_n V_n$ then prove that $\alpha_i = (w, V_i)$ for $i = 1, 2, ..., n$.

(*ii*) Let V be the set of all continuous complex-valued function on the closed unit interval [0, 1]. If $f(t), g(t) \in V$, define by :

$$(f(t), g(t)) = \int_0^1 f(t) \, . \, \overline{g(t)} \, dt.$$

then verify that this defines an inner product on V.

(*b*) (*i*) Define orthogonal complement of subspace of a inner product space, and prove that if W is subspace of inner product space V over F, then W^1 is subspace of V.

(*ii*) If a, b, c are real numbers such that $a > 0$ and $a\lambda^2 + 2b\lambda + c \geq 0$ for all real numbers λ, then prove that :

$$b^2 \leq ac.$$

4. Attempt any *two* of the following : 8

(*a*) If V is finite dimensional vector space over F and $T \in A(V)$ such that the constant term of the minimal polynomial for T is not zero then prove that T is invertible.

(*b*) If V is finite dimensional vector space over F, and $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that for any polynomial $q(X) \in F[X]$, $q(\lambda)$ is a characteristic root of $q(T)$.

(*c*) Let $V = F^{(3)}$ and suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the matrix of T in A(V) in the basis $V_1 = (1, 0, 0)$, $V_2 = (0, 1, 0)$, $V_3 = (0, 0, 1)$. Find matrix of T in the basis :

$$u_1 = (1, 1, 1),\ u_2 = (0, 1, 1),\ u_3 = (0, 0, 1).$$

5. Attempt any *one* of the following : 8

(*a*) If V and W are of dimension *m* and *n*, respectively over F, then prove that Hom(V, W) is of dimension *mn* over F.

(*b*) Prove that every finite dimensional inner product space has an orthonormal set as a basis.

(*c*) Define characteristic root of T in A(V) where V is finite dimensional vector space over F. And prove that characteristic vectors of T belonging to distinct characteristic roots are linearly independent over F.