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B—80—2019

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

MARCH/APRIL, 2019

(CGPA Pattern)

MATHEMATICS

Paper-XIV (MT-304)

(Linear Algebra)

(Wednesday, 27-3-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define internal direct sum of the vector space V.

(*b*) If V is a vector space then define linearly dependent over F.

(*c*) Define orthogonal in vector space.

(*d*) Define an algebraic number.

(*e*) Define characteristic vector of T.

(*f*) Compute the following matrix :

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}^2 .$$

P.T.O.

2. Attempt any *two* of the following : 5 each

(*a*) If v_1, v_2 ...., v_n is a basis of v over F and if w_1, w_2, ..., w_m in V are linearly independent over F, then prove that $m \leq n$.

(*b*) If v_1, v_2 ...., v_n are in V then prove either they are linearly independent or some V_k is a linear combination of the preceding ones, v_1, v_2, ..., v_{k-1}.

(*c*) If F is the field of real numbers, then prove that the vectors (1, 1, 0, 0), (0, 1, –1, 0) and (0, 0, 0, 3) in $F^{(4)}$ are linearly independent over F.

3. Attempt any *two* of the following : 5 each

(*a*) If $u, v \in V$ and $\alpha, \beta \in F$ then prove that :

$$(\alpha u + \beta v, \alpha u + \beta v) = \alpha\overline{\alpha}(u, u) + \alpha\overline{\beta}(u, v) + \overline{\alpha}\beta(v, u) + \beta\overline{\beta}(v, v).$$

(*b*) If V is a finite dimensional inner product space and if W is a subspace of V, then prove that :

$$V = W + W^1,$$

V is the direct sum of W and W^1.

(*c*) In V define the distance $\xi(u, v)$ from u to v by $\xi(u, v) = \| u - v \|$, then prove that :

(*i*) $\xi(u, v) \geq 0$ and $\xi(u, v) = 0$ if and only if $u = v$

(*ii*) $\xi(u, v) = \xi(v, u)$

(*iii*) $\xi(u, v) \leq \xi(v, w) + \xi(w, v)$.

4. Attempt any *two* of the following : 5 each

(*a*) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.

(*b*) If V is a finite-dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

(*c*) Let V be two-dimensional over the field F, or real numbers, with a basis V_1, V_2. Find the characteristic roots and corresponding characteristic vectors for T defined by :

$$V_1T = V_1 + 2V_2, \; V_2T = 3V_1 + 6V_2.$$