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X—32—2019

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) (Regular) EXAMINATION

OCTOBER/NOVEMBER, 2019

(CBCS/New Pattern)

MATHEMATICS

Paper XIV (A)

(Operation Research)

(Thursday, 17-10-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. State the major steps for mathematical formulation of linear programming problem and also determine the optimal product mix for a company makes two kinds of leather belts. Belt A is a high quality belt and belt B is of lower quality. The respective profits are ₹ 4·00 and ₹ 3·00 per belt. Each belt of type A requires twice as much time as a belt of type B, and if all belts were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). Belt A requires a fancy buckle and only 400 buckles per day are available. There are only 700 buckles a day available for belt B. 15

Or

(*a*) Define optimum solution and explain canonical form of linear programming problem. 8

(*b*) Consider the following problem faced by a production planner in a soft drink plant. He has two bottling machines A and B. A is designed for 8-ounce bottles and B for 16-ounce bottles. However, each can be used on both types with some loss of efficiency. The following data is available.

P.T.O.

Machine	8-ounce bottles	16-ounce bottles
A	100/minute	40/minute
B	60/minute	75/minute

Each machine can be run 8-hours per day, 5 days per week. Profit on a 8-ounce bottle is 25 paise and on a 16-ounce bottle is 35 paise. Weekly production of the drink cannot exceed 3,00,000 ounces and the market can absorb 25,000 8-ounce bottles and 7,000 16-ounce bottles per week. The planner wishes to maximize his profit subject, of course, to all the production and marketing restrictions. Formulate this as a LPP. 7

2. Explain Simplex Algorithm for the solution of L.P.P. and obtain all the basic solutions of the following system of linear equations : 15

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5$$

Or

(*a*) Prove that A sufficient condition for a basic feasible solution to an LPP to be an optimum (maximum) is that $z_j - c_j$ for all j for which the column vector $a_j \in$ A is not in the basis B. 8

(*b*) Solve the following system of simultaneous linear equations by using the simplex method : 7

$$x_1 + x_2 = 1$$

$$2x_1 + x_2 = 3.$$

3. Attempt any *two* of the following : 5 each

(*a*) If $x_{ij} = x_{ij}^*$ minimizes $z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$ with $\sum_{i=1}^{n} x_{ij} = 1$, $\sum_{j=1}^{n} x_{ij} = 1$; $x_{ij} = 0$ or 1, then prove that x_{ij}^* also minimize $z^* = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^* x_{ij}$, where $c_{ii}^* = c_{ij} - u_i - v_j$ for all $i^*j = 1, 2, \ldots, n$ and u_i, v_j are some real numbers.

(*b*) Prove that the number of basic variables of the general transportation problem at any stage of feasible solution must be $m + n - 1$.

(*c*) A student has to select one and only one elective in each semester and the same elective should not be selected in different semesters. Due to various reasons, the expected grades in each subject, if selected in different semesters vary and they are given below :

Semester	Analysis	Statistics	Graph Theory	Algebra
I	F	E	D	C
II	E	E	C	C
III	C	D	C	A
IV	B	A	H	H

The grade points are : H = 10, A = 9, B = 8, C = 7, D = 6, E = 5 and F = 4. How will the student select the electives in order to maximize the total expected points and what will be his maximum expected total points ?

(*d*) A company wishes to assign 3 jobs to 3 machines in such a way that each job is assigned to some machine and no machine works on more than one job. The cost of assigning job i to machine j is given by the matrix below :

$$\text{Cost Matrix} \begin{bmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{bmatrix}$$

Draw the associated Network. Formulate the Network LPP and find the minimum cost of making.