This question paper contains **2** printed pages]

X—34—2019

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) (Regular) EXAMINATION

OCTOBER/NOVEMBER, 2019

(CBCS Pattern)

MATHEMATICS

Paper-XIV

(Complex Analysis)

(Thursday, 17-10-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Explain the method to obtain the roots of a non-zero complex number z_0. Hence obtain the square roots of $\sqrt{3}+i$. 15

Or

(*a*) For a non-zero complex number $z = (x, y)$ find its multiplicative inverse. 8

(*b*) Using De-Moivre's formula derive the following identities : 7

(*i*) $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$

(*ii*) $\sin 3\theta = 3\cos^2\sin\theta - \sin^3\theta$

2. Let the function $f(z) = u(r,\theta) + iv(r,\theta)$ be defined throughout some $\in$ neighbourhood of a non-zero point $z_0 = r_0\exp(i\theta_0)$ and suppose that : 15

(*i*) The first-order partial derivatives of the functions u and U with respect to r and θ exists everywhere in the neighbourhood.

(*ii*) Those partial derivatives are continuous at (r_0, θ_0) and satisfy the polar form $ru_r = v_\theta, u_\theta = -r\mathrm{V}_r$ of the Cauchy-Riemann equations at (r_0, θ_0). Then prove that $f'(z_0)$ exists and its value being $f'(z_0) = e^{-i\theta}(u_r + iv_r)$. Hence if $f(z) = \dfrac{1}{z}$ then show that $f'(z) = \dfrac{-1}{z^2}$.

P.T.O.

Or

(*a*) Prove that a composition of continuous functions itself is continuous. 8

(*b*) If $f(z) = \frac{\bar{z}}{z}$ then show that $\lim_{z \to 0} f(z)$ does not exist. 7

3. Attempt any *two* of the following : 5 each

(*a*) For non-zero complex number z, prove that :

$$\log z = \ln r + i(2n\pi + \odot)$$

where

$$r = |z|, n = 0, \pm 1, \ldots\ldots$$

(*b*) Find all values of z such that :

$$e^z = 1 + i.$$

(*c*) Show that :

$$\sinh(z_1 + z_2) = \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2.$$

(*d*) Show that :

$$\log(i^3) \neq 3 \log i.$$