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Y—79—2019

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) (Backlog) EXAMINATION

NOVEMBER/DECEMBER, 2019

(CBCS Pattern)

MATHEMATICS

Paper XIII

(Linear Algebra)

(Monday, 23-12-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *four* of the following : 8

(*a*) Define homomorphism of vector space.

(*b*) Define annihilator of subspace of vector space.

(*c*) Define an algebraic number.

(*d*) Define inner product space.

(*e*) Define an algebra over field.

(*f*) Define range of linear transformation.

2. Attempt any *two* of the following : 8

(*a*) Prove that if $v_1, v_2, v_3 \ldots v_n$ are in V, then either they are linearly independent or some V_k is a linear combination of the preceding one's $v_1, v_2, \ldots v_{k-1}$.

P.T.O.

(*b*) If V is finite dimensional and $v \neq 0 \in V$, then prove that there is an element $f \in \hat{V}$ s.t. $f(v) \neq 0$.

(*c*) Prove that the intersection of two subspaces of V is a subspace of V.

3. Attempt any *one* of the following : 8

(*a*) (*i*) If $u, v \in V$ and $\alpha, \beta \in F$ then prove that :

$$(\alpha u + \beta v, \alpha u + \beta v) = \alpha\bar{\alpha}(u,u) + \alpha\bar{\beta}(u,v) + \bar{\alpha}\beta(v,u) + \beta\bar{\beta}(v,v)$$

(*ii*) If V is a finite dimensional inner product space and W is a subspace of V, then prove that $(W^{\perp})^{\perp} = W$.

(*b*) (*i*) If $u, v \in V$, then prove that $|(u,v)| \leq ||u||.||v||$.

(*ii*) Let V be the set of real functions $y = f(x)$ satisfying $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$, then prove that V is a three dimensional real vector space.

4. Attempt any *two* of the following : 8

(*a*) Prove that the element $\lambda \in F$ is a characteristic roots of $T \in A(V)$ iff for some $v \neq 0$ in V, $vT = \lambda v$

(*b*) Let V be a vector space of continuous functions on [0, 1] and a map $T : V \to R$ by for f in V

$$T(f) = \int_0^1 f(x)dx,$$

show that T is linear transformation

(*c*) Compute the following matrix product :

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & -1 & -1 \end{pmatrix}$$

5. Attempt any *one* of the following : 8

(*a*) If V is finite dimensional and W is subspace of V then prove that W is finite dimensional, dimW $\leq$ dimV and dim (V/W) = dimV – dimW

(*b*) Define an orthonormal set. If $\{w_1, w_2, ... w_m\}$ is an orthonormal set in V, prove that :

$$\sum_{i=1}^{m} |(w_i, v)|^2 \leq ||v||^2 \text{ for any } v \in V \, :$$

(*c*) If V is finite dimensional vector space over F, then prove that T$\in$A(V) is invertible iff the constant term of the minimal polynomial for T is not zero.