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Y—80—2019

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) (Backlog) EXAMINATION

NOVEMBER/DECEMBER, 2019

(CGPA Pattern)

MATHEMATICS

Paper XIV (MT-302)

(Linear Algebra)

(Monday, 23-12-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define Quotient space.

(*b*) State Schwarz inequality.

(*c*) Define Linear combination of vectors.

(*d*) Define norm of a vector.

(*e*) Define algebra over a field.

(*f*) Define characteristic vector.

2. Attempt any *two* of the following : 5 each

(*a*) If V is finite dimensional and W is a subspace of V, then prove that $\hat{W}$ is isomorphic to $\hat{V}/A(W)$.

(*b*) If $v_1, v_2, ..., v_n \in V$ are linearly independent, then prove that every element in their linear span has a unique representation in the form $\lambda_1 v_1 + \lambda_2 v_2 + ... + \lambda_n v_n$ with the $\lambda_i \in F$.

(*c*) If F is a field of real numbers, prove that the vectors (1, 1, 0, 0), (0, 1, –1, 0) and (0, 0, 0, 3) if $F^{(4)}$ are linearly independent over F.

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) If V is a finite-dimensional inner product space and if W is a subspace of V, then prove that V is the direct sum of W and $W^{\perp}$.

(*b*) If V is the set of all continuous, complex valued functions on [0, 1] if for $f(t), g(t) \in V$ we have

$$\left(f(t), g(t)\right) = \int_0^1 f(t)\,\overline{g(t)}\,dt.$$

Then prove that this defines an inner product on V.

(*c*) If V is finite-dimensional inner product space and W is a subspace of V, then prove that $(W^{\perp})^{\perp} = W$.

4. Attempt any *two* of the following : 5 each

(*a*) Prove that, if $\lambda \in F$ is a characteristic root of $T \in A(V)$, then for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.

(*b*) If v is finite-dimensional over F, then $T \in A(v)$ is singular iff there exists a $v \neq 0$ in v such that $vT = 0$.

(*c*) Let v be two-dimensional over the field F of real numbers, with a basis v_1, v_2. Find the characteristic roots and corresponding characteristic vectors for T defined by

$$v_1T = v_1 + v_2,\ v_2T = v_1 - v_2.$$