

This question paper contains 3 printed pages]

Y—96—2019

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) (Backlog) EXAMINATION

OCTOBER/NOVEMBER, 2019

(CBCS Pattern)

MATHEMATICS

Paper XIV

(Complex Analysis)

(Tuesday, 19-11-2019)

Time : 10.00 a.m. to 12.00 noon

Time— Two Hours

Maximum Marks—40

N.B. :— (i) All questions are compulsory.

(ii) All questions carry equal marks.

1. Attempt any *four* of the following (Each of **2** marks) : 8

(a) Show that :

$$\operatorname{Re}(iz) = -\operatorname{Im}z$$

(b) Show that :

(i) $\overline{z + 3i} = z - 3i$

(ii) $\overline{iz} = -i\overline{z}$

(c) Evaluate :

$$\lim_{z \rightarrow \infty} \frac{2z + i}{z + 1}$$

(d) Define Entire function. Give *one* example of it.

(e) Use Euler's formula to show :

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

(f) Find the principal value of $(i)^i$.

P.T.O.

2. Attempt any *two* out of the following (Each of 4 marks) : 8

- (a) Verify that each of the two numbers $z = 1 \pm i$ satisfies the equation $z^2 - 2z + 2 = 0$.
- (b) Compute and reduce the following quantity :

$$\left(\frac{1}{2-3i} \right) \left(\frac{1}{1+i} \right)$$

to a real number.

- (c) If $z_1 = -3 + 2i$ and $z_2 = 1 + 4i$, then which point is closer to origin ?

3. Attempt any *one* of the following (Each of 8 marks) : 8

- (a) (i) If $f(z) = z^2$, then express $f(z) = u(r, \theta) + iv(r, \theta)$.
- (ii) If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D. Then prove that its component function u and v are harmonic in D.
- (b) (i) Show that the function $f(z) = \cosh x \cdot \cos y + i \sinh x \cdot \sin y$ is entire.
- (ii) Suppose that a function $f(z) = u(x, y) + iv(x, y)$ and its conjugate $\overline{f(z)} = u(x, y) - iv(x, y)$ are both analytic in a given domain. Show that $f(z)$ is constant in D.

4. Attempt any *two* of the following (Each of 4 marks) : 8

- (a) Show that, when $n = 0, \pm 1, \pm 2, \dots$

$$(i)^{-2n} = \exp[(4n + 1)\pi].$$

- (b) Show that :

$$2 \sin z_1 \cdot \cos z_2 = \sin(z_1 + z_2) + \sin(z_1 - z_2)$$

- (c) Find all the values of z , such that $e^z = -1$.

5. Attempt any *one* of the following (Each of 8 marks) : 8

(a) Explain the method to find the n th roots of non-zero complex number z_0 .

(b) Suppose that :

$$f(z) = u(x, y) + iv(x, y), \quad z_0 = x_0 + iy_0 \text{ and } w_0 = u_0 + iv_0.$$

Then prove that $\lim_{z \rightarrow z_0} f(z) = w_0$ if and only if $\lim_{(x,y) \rightarrow (x_0,y_0)} u(x, y) = u_0$ and

$$\lim_{(x,y) \rightarrow (x_0,y_0)} v(x, y) = v_0.$$

(c) Show that :

$$(-1 + \sqrt{3}i)^{3/2} = \pm 2\sqrt{2}.$$