

CG-11-2020

WINTER EXAM 2020

Subject Name : RB-10_MATHEMATICS - Metric Spaces – XII (CBCS)_V

Date : 17/03/2021

Duration : 60 min. |

Instruction / सुचना / :-

* Follow the detail instructions given on OMR Sheet

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Q.1

If \mathbb{R}^n is the set of all ordered n-Tuples with the function 'd' defined by $d(x, y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$,
 $\forall x, y \in \mathbb{R}^n$ then d is called as

- A) Usual metric
 B) Discrete metric
 C) Euclidean metric
 D) None of these

A]A
 B]B

C]C
 D]D

Q.2

The diameter of any non-empty subset A, of a metric space (x,d), is defined as $d(A) =$

- A) $\text{Sup } \{d(a,b) : a, b \in A\}$
 B) $\text{inf } \{d(a,b) : a, b \in X\}$
 C) $\text{Sup } \{d(a,x) : x \in X\}$
 D) $\text{inf } \{d(a,x) : x \in X\}$

A]A
 B]B

C]C
 D]D

Q.3

Let (x,d) be any metric space and $a \in X$. Then an open sphere of radius $r > 0$, centered at a is defined as

- A) $\{x \in X : d(x, a) \leq r\}$
 B) $\{x \in X : d(x, a) < r\}$
 C) $\{x \in X : d(x, a) \geq r\}$
 D) $\{x \in X : d(x, a) < r\}$

A]A
 B]B

C]C
 D]D

Q.4 Which of the following statement is not correct ?

- A] In discrete metric space, every singleton set is open
 B] In a usual metric space, the singleton set is not open

C] Both (A) & (B)
 D] Neither (A) nor (B)

Q.5 In any metric space, the set F is closed if

- A] it contains all its limit points
 B] its complement is open

C] Both (A) & (B)
 D] Neither (A) nor (B)

Q.6 If \mathbb{R} is the set of real numbers with usual metric 'd' and $A =]0, 1[$, then the set of all limit point of A is

- A] $]a, b[$
 B] $[a, b[$

C] $]a, b]$
 D] $[a, b]$

Q.7 Every closed sphere is

- A] an open set
 B] a closed set

C] a semi-open set
 D] a semi-closed set

Q.8 If A is any subset of a metric space (x, d), then

- A] A is a closed set
 B] $A = \bar{A}$ if and only if A is closed

C] both (A) & (B)
 D] Neither (A) nor (B)

A subset A of a metric space (x, d) is said to be dense in X if

- A) $A = \emptyset$
 B) $A = \bar{A}$
 C) $A = X$
 D) None of the above

Q.9

A]A
 B]B

C]C
 D]D

Q.10 The set of integers has

- A] a unique limit point
 B] finite number of limit points

C] infinite number of limit points
 D] no limit point

Q.11 The arbitrary union of closed sets

- A] is a closed set
 B] need not be a closed set

C] an open set
 D] None of the above

Q.12

Let (x, d) be any metric space and let $d^1(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \forall x, y \in X$

then,

- A) d^1 is also a metric on X
- B) d and d^1 are equivalent
- C) Both (A) and (B)
- D) None of the above

A]A
B]B

C]C
D]D

Q.13 If Y is a subspace of a metric space (x, d) , then the open sphere $S_r^y(y)$ in (Y, d_Y) is given by

- A) $S_r(y) \cap y$
- B) $S_r(y) \cap x$
- C) $S_r(y) \cap \{y\}$
- D) None of the above

A]A
B]B

C]C
D]D

Q.14 In a metric space (x, d) , which of the following statement is correct?

- A) An empty set \emptyset and the whole set x are both open and closed sets.
- B) Every open sphere is an open set
- C) Every closed sphere is a closed set
- D) All of the above

A]A
B]B

C]C
D]D

Q.15 A point $a \in X$ is an adherent point of a subset A of X if every open sphere centered at 'a' contains a point of A

- A]contains a point of A
- B]not contains a point of A

C]is an empty set
D]None of the above

Q.16 A sequence $\{x_n\}$, of points of a metric space (x, d) is called cauchy sequence if for each $\epsilon > 0$, \exists a positive integer n_0 such that $\forall n, m \geq n_0$

- A) $d(x_n, x_m) < \epsilon$
- B) $d(x_n, x_m) \leq \epsilon$
- C) $d(x_n, x_m) \geq \epsilon$
- D) None of the above

A]A
B]B

C]C
D]D

Q.17 A subspace y , of a complete metric space x , is complete if and only if y is

- A]Open set
- B]Complete set

C]Closed set
D]None of the above

Q.18 "Every complete metric space is of second category" is known as

- A]Cantor's Intersection Theorem
- B]Baire's category Theorem

C]Banach fixed point Theorem
D]None of the above

Q.19 Let (x, d) be a complete metric space and let $\{F_n\}$ be a decreasing sequence of non-empty closed subset of x such that $(d(f_n) \rightarrow 0$ as $n \rightarrow \infty$ then $F = \bigcap_{n=1}^{\infty} F_n$ contains

- A) exactly one point
- B) finite number of points
- C) infinite number of points
- D) None of the above

A]A
B]B

C]C
D]D

Q.20 Which of the following statement is correct?

- A]Every convergent sequence is a cauchy sequence.
- B]Every cauchy sequence is a convergent sequence

C]Both (and ()
D]Neither (A) nor (B)

Q.21 The function $f: (x, d_1) \rightarrow (y, d_2)$ is said to be continuous if it is continuous at

- A) only one point of x
- B) each point of x
- C) finite number of point of x
- D) None of the above

A]A
B]B

C]C
D]D

The function $f: (x, d_1) \rightarrow (y, d_2)$ is continuous if and only if

A) $f^{-1}(G)$ is open in X whenever G is open in Y .

B) $f^{-1}(F)$ is closed whenever F is closed in Y .

C) Both (A) and (B)

D) Neither (A) nor (B)

Q.22	A]A B]B	C]C D]D	114	114
	The constant function $f: (x, d_1) \rightarrow (y, d_2)$ is		114	114
	A) continuous on x			
	B) continuous on y			
	C) not continuous on x			
Q.23	D) not continuous on y			
	A]A B]B	C]C D]D	114	114
	Every uniformly continuous function $f: (x, d_1) \rightarrow (y, d_2)$ is		114	114
	A) continuous on x			
	B) not continuous on x			
	C) not continuous on y			
Q.24	D) none of the above			
	A]A B]B	C]C D]D	114	114
	A function $f: (x, d) \rightarrow (y, d^1)$ is called an isometry if $\forall x, y \in X$		114	114
	A) $d(x, y) = d^1(x, y)$			
	B) $d^1(x, y) = d(x, y)$			
	C) $d(x, y) = d^1(f(x), f(y))$			
Q.25	D) None of the above			
	A]A B]B	C]C D]D	114	114
Q.26	A function $f: (x, d) \rightarrow (x, d)$ is said to be a contraction mapping if \exists a positive real number $\infty < 1$ such that $\forall x, y \in X$		114	114
	A) $d(x, y) < \infty \cdot d(x, y)$			
	B) $d(f(x), f(y)) < \infty \cdot d(x, y)$			
	C) $d(f(x), f(y)) < \infty \cdot d(f(x), y)$			
	D) None of the above			
	A]A B]B	C]C D]D	114	114
Q.27	Any contraction mapping f of a non-empty complete metric space (x, d) into itself has		114	114
	A) a unique fixed point			
	B) two fixed points			
	C) finite number of fixed point			
	D) None of the above			
	A]A B]B	C]C D]D	114	114
Q.28	If $f(x) = x^2$, $0 \leq x \leq 1/3$, is a contraction mapping on $[0, 1/3]$ with		114	114
	A) the usual metric			
	B) Discrete metric			
	C) Euclidean metric			
	D) None of the above			
	A]A B]B	C]C D]D	114	114
Q.29	A subset A of a metric space (X, d) is compact if every open covers of A admits		114	114
	A]a subcover	C]an infinite subcover		
	B]a finite subcover	D]None of the above		
Q.30	Every compact subset F of a metric space (X, d) is		114	114
	A]an empty set	C]a closed set		
	B]an open set	D]None of the above		
Q.31	Every compact subset A of a metric space is		114	114
	A]closed set	C]both (A) and (B)		
	B]bounded	D]Neither (A) nor (B)		
Q.32	continuous image of compact set is		114	114
	A]compact	C]not continuous		
	B]continuous	D]None of the above		
Q.33	A metric space (X, d) is said to have Bolzano-Weierstrass Property if every infinite subset of X has			

A]a fixed point B]a limit point	114	C]a isolated point D]None of the above	114
Q.34 A metric space is sequentially compact, if every convergent sequence has A]an infinite subsequence B]a convergent subsequence	114	C]a subsequence D]none of the above	114
Q.35 Two sets A and B in a metric space (x, d) are said to be separated is A) $A \cap \bar{B} = \phi$ B) $\bar{A} \cap B = \phi$ C) Both (A) and (B) D) None of the above	114	C]C D]D	114
Q.36 Two disjoint set A and B are A]separated B]not separated	114	C]either (or { D]Neither (A) nor (B)	114
Q.37 Which of the following statements are equivalent A) y is connected B) y cannot be expressed as disjoint union of two non-empty closed sets in y . C) ϕ and y are the only sets which are both open and closed in y . D) All of the above	114	C]C D]D	114
Q.38 The union of two connected sets is connected if it has A]empty intersection B]non-empty intersection	114	C]a single point D]none of these	114
Q.39 Continuous image of connected set is A]connected B]not connected	114	C]separated D]None of the above	114
In the Euclidean space \mathbb{R}^2 , $D = \{(x, y) : x \neq 0, y = \sin 1/x\}$ is A) connected B) not connected C) compact D) None of the above	114	C]C D]D	114
Q.40 A]A B]B		C]C D]D	