CG-11-2020

WINTER EXAM 2020

Subject Name : RB-18_MATHEMATICS - Linear Algebra – XIII (CBCS)_V_18-03-2021

Date : 18/03/2021 Duration : 60 min.

Instruction / सुचना / :-

* Follow the detail instructions given on OMR Sheet

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Q.1

If V is a vector space over a field F, then for all ∝∈ F and for all v, w ∈ V, which of the following is correct ?

a) (v+w)∝ = v∝ +w∝

b) (v+w)∝ = ∝v +∝w

c) ∝(v+w) = v∝ +w∝

d) ∝(v+w) = ∝v +∝w

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Q.2

If F is a field and K is a field which contains F as a subfield, then which of the following is a vector space ?

A]F is a vector space over K

B]K is a vector space over F

C]Both F is a vector space over K and K is a vector space over F

D]Neither F is a vector space over K nor K is a vector space over F

Q.3

Which of the following is the criterion for a nonempty subset W of a vector space V over a field F to be a subspace of V ?

a) $\propto_1 \beta \in \mathbf{F}$ & $\mathbf{w_1, w_2 \in W \Rightarrow \propto w_1 + \beta W_2 \in W}$

b) $\propto_1 \beta \in \mathbf{F}$ & $\mathbf{w_1, w_2 \in W \Rightarrow \propto w_1 + \beta W_2 \in V}$

c) $\propto_1 \beta \mathbf{CF}$ & $\mathbf{w_1, w_2 \in W \Rightarrow \propto w_1 + \beta W_2 \in F}$

d) $\propto_1 \beta \in \mathbf{F}$ & $\mathbf{w_1, w_2 \in W \Rightarrow \propto w_1 + \beta W_2 \in V - W}$

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Q.4

What is an isomorphism?

A]Just a homomorphism

B]A one – to – one homomorphism

C]An onto homomorphism

D]A one-to-one onto mapping

Q.5

If U and V are vector spaces over F and T is a homomorphism of U and V, the what is kernel of T ?

a) $\{\mathbf{u \in U}$ | uT = O}

b) $\{\mathbf{u \in U}$ | uT = 1}

c) $\{\mathbf{v \in V}$ | oT = v}

d) $\{\mathbf{v \in V}$ | 1T = v}

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Q.6

If V is a vector space over F, the for all∝$\in \mathbf{F}$, $\mathbf{v \in V}$, $\propto (-\mathbf{v})$ = ?

a) $\propto \mathbf{V}$

b) $\propto^{-1}$ V

c) $\propto \mathbf{V}^{-1}$

d) $-$ (2V)

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Q.7

If V is a vector space over F and If W is a subspace of V, then what is the quotient space $V\ /\ W$'

a) $\{\mathbf{v + W}$ | v∈V}

b) $\{\mathbf{v - W}$ | v∈V}

c) $\{\mathbf{v.W}$ | v∈V}

d) $\{\mathbf{v \div W}$ | v∈V}

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Q.8

When is a vector space V over F said to be finite – dimensional?

A]If there exists any finite set S such that L(S) = V

B]If there exists any infinite set S such that L(S) = V

C]If there exists any finite subset S of V such that L(S) = V

D]If there exist any infinite subset S of V such that L(S) = V

Q.9

If V is a vector space over F, the when are V_1, V_2 ......, V_n ∈V said to be linearly independent over F ?

a) If, and only if, there exist elements λ_1, λ_2, ..., $\lambda_n \in F$, not all of them zero (i.e., at least one of them is non-zero) such that $\lambda_1 V_1 + \lambda_2 V_2 + \ldots\ldots + \lambda_n + V_n = 0$

b) If, and only if, there exist elements λ_1, λ_2, ..., $\lambda_n \in F$, not all of them zero (i.e., at least one of them is non-zero) such that $\lambda_1 V_1 + \lambda_2 V_2 + \ldots\ldots + \lambda_n + V_n = 1$

c) Whenever $\lambda_1 V_1 + \lambda_2 V_2 + \ldots\ldots + \lambda_n + V_n = 0$ each $\lambda_i = 0$

d) Whenever $\lambda_1 V_1 + \lambda_2 V_2 + \ldots\ldots + \lambda_n + V_n = 0$ each $\lambda_i = 1$

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Q.10

If a basis of a vector space V over F contains n number of elements and any other subset of linearly independent vectors contains m number of elements, the how are n & m related?

a) m = n

b) $m \geq$ n

c) m ≤ n

d) m ≠ n

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Q.11

If V is a finite-dimensional vector space of dimension m over F, then what is the dimension of Hom $(V_1 V')$?

a) m^2

b) 2m

c) m^m

d) m

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Q.12

If V is a vector space over F, then what its dual space?

a) Hom $(V_1 V)$

b) Hom $(V_1 F)$

c) Hom $(F_1 V)$

d) Hom $(F_1 F)$

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Q.13

If W is a subspace of a vector space V over F, the A (A (W)) =?

a) W

b) A(W)

c) V – W

d) $V/_{W}$

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Q.14

If W is a subspace of a vector space V over F, then its annihilator A (W) is subspace of which vector space ?

A]V

B]F

C]Dual space of V

D]Second dual space of V

Q.15

If V is an inner product space over F and u∈V is such that $(u_1 u)$ = 0, the u =?

a) 0

b) 1

c) −1

d) Arbitrary (any vector)

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Q.16

If V is an inner product space over F, the for all u, v, w∈V and for all $\propto_1 \beta \in F$, $(\propto u + \beta V, w)$ =?

a) $(\propto u, \propto w)$ + $(\beta V, \beta w)$

b) $\beta (u, w)$ + $\propto (V, w)$

c) $(u, \propto w)$ + $(V, \beta w)$

d) $\propto (u, w)$ + $\beta (V, w)$

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Q.17

If V is an inner product space over F, then for all u∈V and for all ∝∈ F, $\|\propto u\|$ = ?

a) ∝ . u
b) $\|\propto\| \cdot \| u \|$
c) | ∝ | · | u |
d) | ∝ |· $\| u \|$

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Q.18

Which of the following is Schwarz inequality for $\boldsymbol{u}$, $\boldsymbol{v}$ belonging to an inner product space V over F ?

a) $|(\boldsymbol{u}, \boldsymbol{v})| \geq \| \gamma \| \cdot \| v \|$
b) $|(\boldsymbol{u}, \boldsymbol{v})| > \| \gamma \| \cdot \| v \|$
c) $|(\boldsymbol{u}, \boldsymbol{v})| \leq \| \gamma \| \cdot \| v \|$
d) $|(\boldsymbol{u}, \boldsymbol{v})| < \| \gamma \| \cdot \| v \|$

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Q.19

In an inner product space V over F, when is a vector $\boldsymbol{u}$ said to be orthogonal to a vector V ?

a) $(\boldsymbol{u}, \boldsymbol{v}) = \boldsymbol{0}$
b) $(\boldsymbol{u}, \boldsymbol{v}) > \boldsymbol{0}$
c) $(\boldsymbol{u}, \boldsymbol{v}) < \boldsymbol{0}$
d) $(\boldsymbol{u}, \boldsymbol{v}) \neq \boldsymbol{0}$

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Q.20

If W is a subspace of an inner product space V over a field F, then what is the orthogonal complement of W ?

a) $\{x \in V \mid (x, \omega) = 1$, for all $\omega \epsilon W\}$
b) $\{x \in V \mid (x, \omega) = \boldsymbol{0}$, for all $\omega \epsilon W\}$
c) $\{x \in V \mid (x, \omega) = < \boldsymbol{0}$ for all $\omega \epsilon W\}$
d) $\{x \in V \mid (x, \omega) = > \boldsymbol{0}$ for all $\omega \epsilon W\}$

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Q.21

How is every orthonormal set of vectors in an inner product space V over a field F ?

A]Empty
B]Linearly dependent
C]Linearly independent
D]Equal to V

Q.22

If V is a finite-dimensional inner product space over a field F and W is any subspace of V, then V is always equal to which of the following?

A]W
B]Orthogonal complement of W
C]Direct sum of W and its orthogonal complement
D]Cannot be obtained for W & its orthogonal complement

Q.23

If u and v are any two vectors in a finite-dimensional inner product space V over a field F, then what is the formula $\| u + v \|^2 + \| u - v \|^2 = 2\,(\| u \|^2 + \| v \|^2)$ called?

a) Triangle Law
b) Quadrilateral Law
c) Square Law
d) Parallelogram Law

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Q.24

If F is f field and K is a field that contains F, then what is K is called?

A]Subfield of F
B]Subspace of F
C]Extension of F
D]Quotient space of F

Q.25

If K is an extension of a field F, then what is degree of K over F ?

a) Order of K
b) Order of $K/_{F}$
c) Order of K – F
d) Dimension of K as a vector space over F

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Q.26

When is an extension K of a field F said to be algebraic extension?

A]If no element of K is algebraic over F
B]If every element of K is algebraic over F
C]If no element of F is algebraic over K
D]If every element of F is algebraic over K

Q.27

What is an algebraic number?

A]A real number which is algebraic over the field of rational numbers
C]A complex number which is algebraic over the field of rational numbers

B]An imaginary number which is algebraic over the field of rational numbers

D]A complex number which is not algebraic over the field of rational numbers.

Q.28

If V is a vector space over a field F then for any T_1, T_2 ∈ Hom $(v_1 v)$, how is the mapping $T_1 + T_2$ defined ?

a) V $(T_1 + T_2)$ = $T_1 + T_2$, ⊬ v ∈V
b) V $(T_1 + T_2)$ = $VT_1 + T_2$, ⊬ v ∈V
c) V $(T_1 + T_2)$ = $T_1 + VT_2$, ⊬ v ∈V
d) V $(T_1 + T_2)$ = $VT_1 + VT_2$, ⊬ v ∈V

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Q.29

If V is a vector space over a field F, then what are the elements of A (v) called)?

A]Functions

B]Functionals

C]Isomorphisms

D]Linear Transformations

Q.30

For a vector space V over a field F, when is a T ∈ A (V) said to be singular?

a) If it is regular
b) If it is non invertible
c) If it is invertible
d) If it is both left-invertible & right-invertible

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Q.31

If V is a finite-dimensional vector space over a field F and T ∈ A (V) is invertible, then what is the constant term in the minimal polynomial for T?

a) 0
b) 1
c) -1
d) Non-Zero

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Q.32

If S, T ∈ A (V) are such that S is rugular, then which of the following is true?

a) r (T) = r (STS^7)
b) r (T) $>$ r (STS^7)
c) r (T) $<$ r (STS^7)
d) r (T) $\neq$ r (STS^7)

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Q.33

For a finite-dimensional vector space V over a field F, and for T∈ A (V), when is λ ∈ F said to be a characteristic root of T?

a) If λ −T is invertibale
b) If λ −T is singular
c) If λ + T is invertibale
d) If λ + T is singular

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Q.34

If λ ∈F is a characteristic root of T∈ A(V), then which of the follwing is guaranteed?

a) λ is a root of minimal polynomial of T
b) λ is a root of maximal polynomial of T
c) λ is not a root of minimal polynomial of T
d) λ is not a root of maximal polynomial of T

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Q.35

What is characteristic vector of a linear transformation belonging to a characteristic root λ of T?

a) Any vector V such that VT = V
b) Any vector V such that VT = λ V
c) Any non-zero vector V such that VT = V
d) Any non-zero vector V such that VT = λ V

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Q.36

If $\lambda_1 , \lambda_2 \dots\dots , \lambda_n$ are distinct characteristic roots of T∈ A (v) and If $V_1, V_2 \dots\dots , V_n$ are charucteristic vectors of T belonging to charucteristic roots$\lambda_1 , \lambda_2, \dots\dots , \lambda_n$. respectively, then how are $V_1, V_2, \dots\dots , V_n$?

a) Linearly dependent
b) Linearly independent
c) Mutually orthogonal
d) Orthonormal

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Q.37

If V is n-dimensional vector space over F, then for T∈ A (v), what is the order of matirx T in any basis of V?

a) n
b) $\mathbf{n \times n}$
c) $\mathbf{n^2}$
d) 2n

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Q.38

If V is a n-dimensional vector space over a field F; $V_1, V_2 \dots\dots , V_n$ is a basis of V over F and for T∈ A (v), If V_iT = $\sum \propto i_j \; V_j$, then what is the matrix of T in the basis $V_1, V_2 \dots\dots , V_n$?

a) $(\propto i)$
b) $(\propto_j)$
c) $(\propto i_j)$
d) $(\propto j_i)$

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Q.39

If V is a n-dimensional vector space over a field F; $V_1, V_2 \dots\dots , V_n$ is a basis of V over F and for T∈ A(v), M (T) is the matrix of T in the basis $V_1, V_2 \dots\dots , V_n$, then how is the mapping T→m(T) ?

a) Only vector space homomorphism of A(v) onto F_n
b) An algebra isomorphism of A(V) onto F_n
c) Only onto
d) Only one – to – one

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Q.40

If v is a n-dimensional vector space over a field F and if T∈A (V) has a matrix, m_1 (T) in the basis $V_1, V_2 \dots\dots , V_n$of V and matrix m_2(T) in the basis $\omega_1, \omega_2 \dots\dots , \omega_n$ of V, the which of the following is correct?

a) m_2(T)= $(m_1(T)^{-1}$, for some matrix C∈ F_n
b) m_1(T)= $(m_2(T)$
c) m_2(T)= $(m_1(T)^2$
d) m_1(T) and $(m_2(T)$ are not related.

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