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BF—54—2016

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Sixth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

(Old Course)

MATHEMATICS

Paper XVII (MT-305)

(Partial Differential Equations)

(Monday, 17-10-2016) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Form the partial differential equation from :

$$x^2 + y^2 + (z - c)^2 = a^2 .$$

(*b*) Solve the partial differential equation :

$$x^2 p + y^2 q + z^2 = 0$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

(*c*) Solve :

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = 0 .$$

P.T.O.

(*d*) Solve the partial differential equation :

$$pq + p + q = 0$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

(*e*) Write one-dimensional heat flow equation and two-dimensional heat flow equation.

(*f*) State the Laplace equation in polar co-ordinates.

2. Attempt any *two* of the following : 5 each

(*a*) Discuss the method to solve Lagrange's linear equation of the type :

$$Pp + Qq = R$$

where P, Q, R are functions of

$$x, y, z \text{ and } p = \frac{\partial z}{\partial x}, \; q = \frac{\partial z}{\partial y}.$$

(*b*) Solve :

$$\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y).$$

(*c*) Find the general solution of :

$$x(z^2 - y^2)\frac{\partial z}{\partial x} + y(x^2 - z^2)\frac{\partial z}{\partial y} = z(y^2 - x^2).$$

3. Attempt any *two* of the following : 5 each

(*a*) Explain Monge's method to solve the non-linear equation of second order :

$$Rr + Ss + Tt = V$$

where R, S, T, V are functions of x, y, z, p and q :

$$r = \frac{\partial^2 f}{\partial x^2},\ s = \frac{\partial^2 f}{\partial x \partial y},\ t = \frac{\partial^2 f}{\partial y^2}.$$

(*b*) Explain the method to solve the equation of the type $F(z, p, q) = 0$ i.e. equations not containing x and y where

$$p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}.$$

(*c*) Solve :

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x.$$

4. Attempt any *two* of the following : 5 each

(*a*) Solve the wave equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

by D' Alembert's method.

(*b*) A rod of length l with insulated sides is initially at a uniform temperature u. Its ends are suddenly cooled to 0ºC and are kept at that temp. Prove that the temperature function $u(x, t)$ is given by :

$$u(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \, e^{-\frac{c^2\pi^2 n^2 t}{l^2}}$$

where b_n is determined from the equation :

$$U_0 = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}.$$

(*c*) Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

which satisfies the conditions :

$$u(0, y) = u(l, y) = u(x, 0) = 0 \text{ and } u(x, a) = \frac{\sin n\pi x}{l}.$$