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BF—55—2016

FACULTY OF ARTS/SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

(New Course)

MATHEMATICS

Paper XVII (MT-305)

(Integral Transforms)

(Monday, 17-10-2016)

Time : 10.00 a.m. to 12.00 noon

Time—2 Hours

Maximum Marks—40

N.B. :— (i) All questions are compulsory.

(ii) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(i) Define the Laplace transform of the function $f(t)$.

(ii) Find the Laplace transform of $t^3 \cdot \delta(t - 4)$.

(iii) Find the inverse Laplace transform of $\frac{s+2}{(s+2)^2 - 25}$.

(iv) State the convolution theorem for inverse Laplace transform.

(v) State the shifting property of Fourier transform of $f(x)$.

(vi) Define the Fourier cosine transform of $f(x)$.

2. Attempt any *two* of the following : 5 each

(i) If $L[f(t)] = F(s)$, then prove that :

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} [F(s)].$$

P.T.O.

(ii) Let $f(t)$ be a periodic function with period T , then prove that :

$$L[f(t)] = \frac{\int_0^T e^{-st} f(t) dt}{1 - e^{-sT}}.$$

(iii) Find the Laplace transform of $e^{-4t} \cdot \frac{\sin 3t}{t}$.

3. Attempt any *two* of the following : 5 each

(i) Find the inverse Laplace transform of $\frac{s-1}{s^2-6s+25}$.

(ii) Using Laplace transforms, find the solution of initial value problem :

$$y'' + 9y = 6 \cos 3t$$

where $y(0) = 2, y'(0) = 0$.

(iii) Find the inverse Laplace transform of $\frac{s^2+3}{s(s^2+9)}$.

4. Attempt any *two* of the following : 5 each

(i) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that :

$$F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right).$$

(ii) Derive the Fourier cosine integral :

$$f(x) = \frac{2}{\pi} \int_0^\infty \cos ux \, du \int_0^\infty f(t) \cos ut \, dt.$$

(iii) Find the Fourier sine transform of $f(x) = e^{-ax}$.