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BF—68/69—2016

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Sixth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

MATHEMATICS

Paper XVIII (A)

(Topology)

Or

Paper XVIII (306-B)

[Mechanics—II (Dynamics)]

(Wednesday, 19-10-2016) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

Paper XVIII (A)

(Topology)

N.B. :—(*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define countably infinite set.

(*b*) State the well-ordering theorem.

(*c*) Define topological space.

(*d*) Define the order topology.

(*e*) Define closure of a set.

(*f*) Define compact space.

2. Attempt any *two* of the following : 5 each

(*a*) Let X be a set; let **B** be a basis for a topology τ on X. Then prove that τ equals the collection of all unions of elements of **B**.

P.T.O.

(*b*) Let X be a set, τ_f be a collection of all subsets U of X such that X – U either is finite or is all of X. Then show that τ_f is a topology on X.

(*c*) Prove that the lower limit topology τ' on R is strictly finer than the standard topology τ.

3. Attempt any *two* of the following : 5 each

(*a*) If **B** is a basis for the topology of X, then prove that the collection :

$$\mathbf{B}_Y = \{B \cap Y \mid B \in \mathbf{B}\}$$

is a basis for the subspace topology on Y.

(*b*) If **B** is a basis for the topology of X, and **C** is a basis for the topology of Y, then the collection

$$D = \{B \times C \mid B \in \mathbf{B},\ C \in \mathbf{C}\}$$

is a basis for the topology of XXY.

(*c*) Let Y = [0, 1] be a subset of real line R, show that Y is a subspace topology of R.

4. Attempt any *two* of the following : 5 each

(*a*) Let Y be a subspace of X. Then prove that a set A is closed in Y if and only if it equals the intersection of a closed subset of X with Y.

(*b*) If the sets C and D form a separation of X, and if Y is a connected subset of X, then prove that Y lies entirely within C or D.

(*c*) Let A and B are subsets of X. Show that :

$$\overline{A \cap B} = \bar{A} \cap \bar{B}.$$

OR

Paper XVIII (306-B)

[Mechanics—II (Dynamics)]

N.B. :—(*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define Acceleration.

(*b*) Write the components of velocity and acceleration along rectangular Cartesian axes.

(*c*) Define moment of momentum.

(*d*) Write the unit of power in C.G.S. system and M.K.S. system.

(*e*) Define Horizontal range of projectile.

(*f*) Define trajectory.

2. Attempt any *two* of the following : 5 each

(*a*) Explain angular speed and angular velocity.

(*b*) Find the radial and transverse components of acceleration.

(*c*) Prove that if the tangential and normal accelerations of a particle describing a plane curve be constant throughout the motion, the angle ψ through which the direction of motion turns in time t is given by :

$$\psi = A \log(1 + Bt).$$

3. Attempt any *two* of the following : 5 each

(*a*) Prove that the principle of conservation of linear momentum.

(*b*) Prove that in a conservative field of force, the sum of kinetic energy and potential energy of a particle at every point, is constant.

(*c*) Find the work done by the force

$$\vec{F} = 2xi + 2yj$$

in moving a particle from P(1, 2) to Q(3, 2).

4. Attempt any *two* of the following : 5 each

(*a*) Find the vertex and the latus rectum of the parabola.

(*b*) Prove that the relation :

$$t_1 . t_2 = \frac{2R}{g}.$$

(*c*) T is the time of flight of a bullet when the horizontal range is R. Prove that the inclination of the direction of projection with the horizontal is :

$$\tan^{-1}\left[\frac{gT^2}{2R}\right].$$