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# R—42—2017

## FACULTY OF ARTS/SCIENCE

## B.A./B.Sc. (Third Year) (Sixth Semester) EXAMINATION

## MARCH/APRIL, 2017

MATHEMATICS

Paper XVI (MT-304)

(Numerical Analysis)

**(Monday, 27-3-2017)** **Time : 10.00 a.m. to 12.00 noon**

*Time—Two Hours* *Maximum Marks*—40

*N.B.* :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of non-scientific/non-programmable calculator is allowed.

1. Attempt any *five* of the following : 2 each

(*a*) Evaluate :

$$\Delta^2 x^3.$$

(*b*) Construct difference table for values of $y = f(x)$ given :

| $x$ | $y$ |
|---|---|
| 0 | 1 |
| 1 | 9 |
| 2 | 25 |
| 3 | 55 |

(*c*) Define central sum operator σ.

(*d*) Prove that :

$$\delta = \nabla E^{1/2}.$$

(*e*) State general quadrature formula.

(*f*) State Simpson's $\frac{1}{3}$rd rule.

P.T.O.

2. Attempt any *two* of the following : 5 each

(*a*) Prove the Newton-Gregory formula for backward interpolation using polynomial in $x$ of degree $n$.

$$P_n(x) = A_0 + (x - a - nh) + A_2(x - a - nh)$$

$$(x - a - nh + h) + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$+ A_n(x - a - nh)(x - a - nh + h) \ldots\ldots\ldots (x - a - h).$$

(*b*) Prove that :

(*i*) $\Delta(u_x v_x) = u_{x+1}\ \Delta v_x + v_x \Delta ux$

(*ii*) $\Delta(u_x v_x) = u_x\ \Delta v_x + v_{x+1} \Delta ux$ .

(*c*) Evalaute :

$$\frac{\Delta^2}{E} x^3.$$

3. Attempt any *two* of the following : 5 each

(*a*) Prove that the $n$th divided difference can be expressed as the quotient of two determinants, each of order $n + 1$.

(*b*) Find the value of $f(15)$ from the following table :

| $x$ | $f(x)$ |
|---|---|
| 4 | 48 |
| 5 | 100 |
| 7 | 294 |
| 10 | 900 |
| 11 | 1210 |
| 13 | 2028 |

(*c*) Given :

$\log_{10} 654 = 2.8156$,

$\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$,

$\log_{10} 661 = 2.8202$, find $\log_{10} 656$.

4. Attempt any *two* of the following : 5 each

(*a*) Prove that trapezoidal rule as approximate quadrature formula.

(*b*) Evaluate :

$$\int_0^{\frac{1}{2}\pi} \sin x \, dx$$

by using Simpson's $\frac{1}{3}$rd rule.

Given :

$\sin 0 = 0$, $\sin \frac{\pi}{20} = 0.1564$, $\sin \frac{\pi}{10} = 0.3090$

$\sin \frac{3\pi}{20} = 0.4540$, $\sin \frac{\pi}{5} = 0.5878$

$\sin \frac{\pi}{4} = 0.7071$, $\sin \frac{3\pi}{10} = 0.8090$,

$\sin \frac{7\pi}{20} = 0.8910$, $\sin \frac{2\pi}{5} = 0.9511$,

$\sin \frac{9\pi}{20} = 0.9877$, $\sin \frac{\pi}{2} = 1.0000$.

P.T.O.

(*c*) Given :

$$\frac{dy}{dx} = \frac{y - x}{y + x}$$

$$= f(x, y)$$

with $y = 1$ when $x = 0$. Find approximately the value of $y$ for $x = 0.1$ by Picard's method (upto second approximation only).