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R—55—2017

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

MARCH/APRIL, 2017

(New Course)

MATHEMATICS

Paper XVII

(Integral Transforms)

(Thursday, 30-3-2017)

Time : 10.00 a.m. to 12.00 noon

Time—2 Hours

Maximum Marks—40

N.B. :— (i) All questions are compulsory.

(ii) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 marks each

(a) Find the Laplace transform of $t \sinh at$.

(b) Define unit step function.

(c) Find the inverse Laplace transform of :

$$\frac{1}{s^2 + 16}$$

(d) State the second shifting property for inverse Laplace transform.

(e) Find the Fourier cosine transform of :

$$f(x) = 1, \quad 0 < x < a \\ = 0, \quad x > a.$$

(f) Write the formulae for Fourier sine transform of $f(x)$ and inverse Fourier sine transform of $F(s)$.

P.T.O.

2. Attempt any *two* of the following :

5 marks each

(a) If

$$L(f(t)) = F(s),$$

then prove that

$$L(t^n f(t)) = (-1)^n \frac{d^n}{ds^n} [F(s)]$$

(b) If

$$L[f_1(t)] = F_1(s) \text{ and}$$

$$L[f_2(t)] = F_2(s),$$

then prove that

$$L \left\{ \int_0^t f_1(x) \cdot f_2(t-x) dx \right\} = F_1(s) \cdot F_2(s)$$

(c) Find the Laplace transform of the waveform :

$$f(t) = \left(\frac{2t}{3} \right), 0 \leq t \leq 3.$$

3. Attempt any *two* of the following :

5 marks each

(a) Find the inverse Laplace transform of :

$$\frac{s+8}{s^2+4s+5}.$$

(b) Find the inverse Laplace transform of :

$$\frac{s^2}{(s^2+a^2)(s^2+b^2)}.$$

- (c) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 4y = 0, \text{ where } y = 0 \text{ and } \frac{dy}{dx} = -6 \text{ at } x = 0,$$

using Laplace transform.

4. Attempt any *two* of the following : 5 marks each

- (a) Prove that the Fourier cosine integral is given by :

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \cos ux \, du \int_0^{\infty} f(t) \cos ut \, dt.$$

- (b) State and prove the ‘change of scale property’ for the Fourier transform.
- (c) Find the Fourier transform of :

$$f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}.$$