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**V—53—2017**

**FACULTY OF SCIENCE**

**B.Sc. (Third Year) (Sixth Semester) EXAMINATION**

**OCTOBER/NOVEMBER, 2017**

**(New Course)**

**MATHEMATICS**

**Paper-XVII**

**(Integral Transform)**

**(Saturday, 14-10-2017)**

**Time : 10.00 a.m. to 12.00 noon**

*Time—2 Hours*

*Maximum Marks—40*

*N.B. :— (i) All questions are compulsory.*

*(ii) Figures to the right indicate full marks.*

1. Attempt any *five* of the following : 2 each

(a) Find the Laplace transform of  $f(t) = 1$ .

(b) Define the unit impulse function.

(c) Find the inverse Laplace transform of  $\frac{S}{S^2 - 25}$ .

(d) Find  $L^{-1} \left[ \frac{1}{(S+2)^3} \right]$ .

(e) Find the Fourier sine transform of

$$f(x) = \begin{cases} 1 & 0 < x < a \\ 0 & x > a \end{cases}$$

(f) Write the formulae for Fourier cosine transform and inverse Fourier cosine transform.

P.T.O.

2. Attempt any *two* of the following :

5 each

(a) Prove that :

$$\begin{aligned} L[f^n(t)] &= S^n L[f(t)] - S^{n-1} f(0) \\ &\quad - S^{n-2} f'(0) - S^{n-3} f''(0) - \dots - f^{n-1}(0) \end{aligned}$$

(b) Let  $f(t)$  be a periodic function with period  $T$ , then prove :

$$L[f(t)] = \frac{\int_0^T e^{-st} f(t) dt}{1 - e^{-sT}}$$

(c) Obtain the Laplace transform of  $t^2 \cdot e^t \sin 4t$ .

3. Attempt any *two* of the following :

5 each

(a) Find the inverse Laplace transform of  $\frac{S^2 + 3}{S(S^2 + 9)}$ .

(b) Find :  $L^{-1} \left[ \frac{S+2}{S^2 - 4S + 13} \right]$ .

(c) Solve the differential equation

$$\frac{d^2 y}{dx^2} + y = 0$$

where  $y = 1, \frac{dy}{dx} = 1$  at  $x = 0$  using Laplace transform.

4. Attempt any *two* of the following :

5 each

(a) Prove that, Fourier sine integral is given by,

$$f(x) = \frac{2}{\pi} \int_0^\infty \sin ux \, dx \int_0^\infty f(t) \sin ut \, dt.$$

(b) State and prove shifting property for Fourier transform.

(c) Find the Fourier transform of :

$$f(x) = \begin{cases} 1 - x^2 & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}.$$