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V—66—2017

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Sixth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2017

MATHEMATICS

Paper-XVIII (A)

(Topology)

(Saturday, 11-11-2017) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Define restriction mapping

(*b*) Define equivalence relation

(*c*) Define projection mapping

(*d*) Define Limit point of a set

(*e*) Define Topological space

(*f*) Define separation.

2. Attempt any *two* of the following : 5 each

(*a*) Let X be a set, let **B** be a basis for a topology **T** on X. Then prove that **T** equals the collection of all unions of elements of **B**.

(*b*) Define lower limit topology also prove that the lower limit topology **T'** on R is strictly finer than the standard topology.

(*c*) If :

$X = \{a, b, c\}$ let $\mathbf{T}_1 = \{\phi, X, \{a\} \{a,b\}\}$ and $\mathbf{T}_2 = \{\phi, X \{a\} \{b,c\}\}$ then find the smallest topology containing $\mathbf{T}_1$ and $\mathbf{T}_2$ and the largest topology contained in $\mathbf{T}_1$ and $\mathbf{T}_2$.

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) Prove that, if A is a subspace of X and B is a subspace of Y, then the product topology on A × B is the same as the topology A × B inherits as a subspace of X × Y.

(*b*) If **B** is a basis for the topology of X, then prove that the collection

$$\mathbf{B}_Y = \{B \cap Y \mid B \in \mathbf{B}\} \text{ is a basis}$$

for the subspace topology on Y.

(*c*) Consider Y be the subset of R, given by

$$Y = [0.1) \cup \{2\}$$

then show that {2} is open in Y when Y is a subspace topology but not open when Y is order topological space.

4. Attempt any *two* of the following : 5 each

(*a*) Prove that, let Y be a subspace of X. Then a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

(*b*) Let X and Y be topological spaces : Let $f : X \to Y$. Then prove that the following are equivalent :

(*i*) f is continuous.

(*ii*) For every subset A of X, one has $f(\bar{A}) \subset \overline{f(A)}$

(*iii*) For every closed set B in Y, the set f^{-1} (B) is closed in X.

(*c*) Let $A \subset X$ and $B \subset Y$, then show that in the space X × Y,

$$\overline{A \times B} = \bar{A} \times \bar{B}.$$