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AO—52—2018

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

MARCH/APRIL, 2018

(New Course)

MATHEMATICS

Paper XVII

(Integral Transforms)

(Friday, 23-3-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *five* of the following : 2 each

(*a*) Find the Laplace transform of $f(t) = e^{at}$.

(*b*) Show that :

$$L[u(t-a)] = \frac{e^{-as}}{s}.$$

(*c*) Find :

$$L^{-1}\left[\frac{s}{s^2+16}\right].$$

(*d*) Find the inverse Laplace transform of :

$$\frac{3s}{2s+a}.$$

P.T.O.

(*e*) Find the Fourier sine transform of :

$$f(x) = 1,\ 0 < x < a$$

$$= 0,\ x > a.$$

(*f*) Define Fourier cosine transform of $f(x)$.

2. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$L\left[\int_0^t f(t)\,dt\right] = \frac{1}{s}F(s)$$

where

$$L[f(t)] = F(s).$$

(*b*) State and prove second shifting theorem for Laplace transform.

(*c*) Evaluate :

$$L\left[e^{-4t}\frac{\sin 3t}{t}\right].$$

3. Attempt any *two* of the following : 5 each

(*a*) Find the inverse Laplace transform of :

$$\frac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}.$$

(*b*) Obtain :

$$L^{-1}\left[\frac{3s+1}{(s-1)(s^2+1)}\right].$$

(*c*) Solve the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$, given $y = 2$, $\frac{dy}{dx} = -4$ at $x = 0$. Use Laplace transform.

4. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iux}\, du \int_{-\infty}^{\infty} f(t)\, e^{-iut}\, dt.$$

(*b*) If F(*s*) is the complex Fourier transform of *f*(*x*), then prove that :

$$\mathrm{F}\{f(x) \cos ax\} = \frac{1}{2}[\mathrm{F}(s + a) + \mathrm{F}(s - a)].$$

(*c*) Find the Fourier sine transorm of $f(x) = e^{-ax}$.