W—57—2018

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(New Course)

MATHEMATICS

Paper XVII

(Integral Transforms)

(Monday, 15-10-2018)   **Time : 10.00 a.m. to 12.00 noon**

Time—2 Hours   *Maximum Marks—40*

N.B. :— (i)   *All* questions are compulsory.

(ii)   Figures to the right indicate full marks.

1.   Attempt any *five* of the following :   2 each

(i)   Show that :

$$L[af_1(t) + bf_2(t)] = aL[f_1(t)] + bL[f_2(t)]$$

(ii)   Draw the graph of :

$$u(t - a) - u(t - b)$$

(iii)   Find the inverse Laplace transform of $\dfrac{S}{S^2 - 16}$.

(iv)   State the second shifting property for inverse Laplace transform.

(v)   Write the formulae for Fourier sine transform and inverse Fourier sine transform.

(vi)   State the shifting property for Fourier transforms.

2.   Attempt any *two* of the following :   5 each

(i)   If

$$L[f(t)] = F(s),$$

then prove that :

$$L[t^n \, f(t)] = (-1)^n \cdot \frac{d^n}{ds^n}[F(s)]$$

P.T.O.

(*ii*) If
$$L[f(t)] = F(s),$$
then prove that :
$$L[f(t - a) \ u(t - a)] = e^{-as} \ F(s).$$

(*iii*) Evaluate :
$$L\left[e^{-4t} \cdot \frac{\sin 3t}{t} \right].$$

3. Attempt any *two* of the following : 5 each

(*i*) Find the inverse Laplace transform of :
$$\frac{3S + 1}{(S - 1)(S^2 + 1)}$$

(*ii*) Find the inverse Laplace transform of :
$$\frac{S}{S^2 + 4S + 13}$$

(*iii*) Using the Laplace transform, find the solution of the initial value problem :
$$y'' + 25y = 10 \cos 5t,$$
$$y(0) = 2, \ y'(0) = 0.$$

4. Attempt any *two* of the following : 5 each

(*i*) State and prove the change of scale property for Fourier transforms.

(*ii*) Prove that the Fourier cosine integral is given by :
$$f(x) = \frac{2}{\pi} \int_0^\infty \cos ux \, du \int_0^\infty f(t) \cdot \cos ut \, dt.$$

(*iii*) Find the Fourier transform of :
$$f(x) = \begin{cases} 1 - x^2, & \text{if} \quad |x| \leq 1 \\ 0, & \text{if} \quad |x| > 1 \end{cases}.$$