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RB—105—2022

FACULTY OF HUMANITIES

B.A. (First Year) (First Semester) EXAMINATION

MAY/JUNE, 2022

(CBCS Pattern)

MATHEMATICS

Paper I

(Differential Calculus)

(Tuesday, 21-6-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—50

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Attempt either A or B for Question Nos. **1**, **2**, **3** and **4.**

(*iii*) Figures to the right indicate full marks.

1. (A) Attempt the following :

State and prove Leibnitz's theorem. 10

Or

(B) Attempt the following : 5

(*a*) Find the nth derivative of :

$$y = e^{ax}\cos(bx + c)$$

(*b*) If $y = \sin ax + \cos ax$, prove that : 5

$$y_n = a^n \sqrt{\{1 + (-1)^n \sin 2ax\}}$$

P.T.O.

2. (A) Attempt the following : 10

Let $f(x + h)$ be a function of $h(x$ – being independent of $h)$ which can be expanded in power of h and the expansion be differentiable any number of times. Then prove that :

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \ldots\ldots + \frac{h^n}{n!}f^n(x).$$

Or

(B) Attempt the following :

(*a*) Prove that the equation of the normal to the Astroid $x^{2/3} + y^{2/3} = a^{2/3}$ may be written as : 5

$$x \sin\phi - y\cos\phi + a\cos 2\phi = 0$$

(*b*) Show that in the case of the curve $\beta y^2 = (x + \alpha)^3$, the square of the sub-tangent varies as the subnormal. 5

3. (A) Attempt the following :

(*a*) If a function 'f' is : 7

(*i*) continuous in a closed interval $[a, b]$

(*ii*) derivable in the open interval $]a, b[$ and

(*iii*) $f(a) = f(b)$, then prove that :

There exists at least one value $C \in]a, b[$ such that : $f'(c) = 0$

(*b*) If in a Cauchy's mean value theorem $f(x) = \sqrt{x}$ and $f(x) = \frac{1}{\sqrt{x}}$. Then show that c is a geometric mean between a and b if a, b, are positive). 3

Or

(B) Show that :

$$\frac{V-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2},\ 0 < u < v \text{ and deduce that :}$$

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$ 10

4. (A) Attempt the following : 10

Define Homogeneous function. If $z = f(x, y)$ be a homogeneous function of x, y of degree n, then prove that :

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

Or

(B) Attempt the following :

(*a*) If $u = 3(lx + my + nz)^2 - (x^2 + y^2 + z^2)$ and $l^2 + m^2 + n^2 = 1$, then show that : 5

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

(*b*) Prove that $f_{xy}(0, 0), \neq f_{yx}(0, 0)$ for the function 'f' given by :

$$f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2};\ (x, y) \neq (0, 0)$$

5

$$f(0, 0) = 0.$$

P.T.O.

5. Attempt any *two* of the following : 5

(*a*) Prove that :

cos$h$ $(x + y) = \cos h \cos hy + \sin hx \sin hy$. 5

(*b*) If $y = a \cos (\log x) + b \sin (\log x)$,

Show that :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

(*c*) Prove that for any quadratic function $px^2 + qx + r$, the value of θ in Lagrange's theorem is always $\frac{1}{2}$ whatever p, q, r, a, h may be. 5

(*d*) If $V = \log_e \left(\sin \left\{ \frac{\pi(2x^2 + y^2 + xz)^{\frac{1}{2}}}{2(x^2 + xy + 2yz + z^2)^{\frac{1}{3}}} \right\} \right)$, 5

then find the value of $x\frac{dv}{dx} + y\frac{dv}{dy} + z\frac{dv}{dz}$ when

$x = 0, y = 1, z = 2$.