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RB—123—2022

FACULTY OF HUMANITIES

B.A. (First Year) (First Semester) EXAMINATION

MAY/JUNE, 2022

MATHEMATICS

Paper–II

(Algebra and Trigonometry)

(Thursday, 23-6-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—50

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to right indicate full marks.

1. (A) Prove that matrix multiplication is associative, *i.e.*, if A, B and C are matrices of orders $m \times n$, $n \times p$ and $p \times q$, respectively, then prove that : 10

$$A(BC) = (AB)C$$

Or

(B) Find the inverse of matrix : 10

$$A = \begin{bmatrix} 9 & 5 & 6 \\ 7 & -1 & 8 \\ 3 & 4 & 2 \end{bmatrix}$$

2. (A) Prove that the elementary operations do not alter the rank of a matrix. 10

Or

(B) Reduce to row echelon form the matrix. 10

$$A = \begin{bmatrix} 1 & -2 & -1 & 4 \\ 2 & -4 & 3 & 5 \\ -1 & 2 & 6 & -7 \end{bmatrix}$$

and also find the rank of A.

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3. (A) Prove that a system AX = B of m linear equations in n unknowns is consistent if, and only if, the coefficient matrix A and the augmented matrix [A : B] of the system have the same rank. 10

Or

(B) Solve the system of equations : 10

$$\begin{aligned} x_1 + 2x_2 + 2x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 2 \\ 3x_1 + 2x_2 + 2x_3 &= 3 \\ x_2 + x_3 &= 0 \end{aligned}$$

4. (A) State and prove De Moivre's Theorem. 10

Or

(B) Prove that : 10

$$\cos n\,\theta = \cos^n\theta - \frac{n(n-1)}{1.2}\cos^{n-2}\theta\sin^2\theta + \frac{n(n-1)(n-2)(n-3)}{1.2.3.4}\cos^{n-4}\theta\sin^4\theta + ...$$

5. Answer any *two* of the following : 10

(A) Define an orthogonal matrix and prove that the determinant of an orthogonal matrix is ± 1.

(B) State any *five* elementary operations for matrices with notations for these operations.

(C) Find the characteristic roots of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$

(D) Prove that, for all values of x and y, real or complex,

$$\sin(x + y) = \sin x \cos y + \cos x \sin y.$$

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