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RB—115—2022

FACULTY OF ARTS

B.A. (First Year) (Second Semester) EXAMINATION

MAY/JUNE, 2022

MATHEMATICS

Paper–III

(Calculus)

(Wednesday, 22-06-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—50

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Attempt either (*a*) *or* (*b*) for question Nos. **1, 2, 3** and **4**.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use suitable data, if required.

(*v*) All symbols carry usual meanings.

1. (*a*) Obtain a reduction formula for $\int x^n e^{-x}\, dx$, hence show that the improper integral $\int_0^\infty x^n e^{-x} = n\,!$ where n is any positive integer. 10

Or

(*b*) Evaluate the following integrals :

(*i*) $\int \frac{dx}{x^3(x-1)^2(x+1)}$

(*ii*) $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$

2. (*a*) Prove that : 10

$$\int x^m (a + bx^n)^p dx = \frac{x^{m+1}(a+bx^n)^p}{np+m+1} + \frac{pna}{np+m+1} \int x^m (a + bx^n)^{p-1} dx$$

P.T.O.

Or

(*b*) Discuss the method to integrate $\frac{1}{\sqrt{ax^2+bx+c}}$. (both cases when a is positive and a – is negative).

3. (*a*) Prove that : 10

$$\int \sin^m x \cos^n x \, dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{(n-1)}{m+n} \int \sin^m x \cos^{n-2} x dx.$$

Or

(*b*) Show that : 10

$$\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

and hence evaluate $\int \sec^7 \theta d\theta$.

4. (*a*) Prove that :

(*i*) $\int_a^b f(x)dx \times \int_c^d g(y)dy = \int_a^b \int_c^d f(x)g(y)dx\,dy$.

(*ii*) Define gamma function and show that :

$$\overline{|(n)} = \int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx$$

Or

(*b*) Find the value of $\overline{\left|\frac{1}{2}\right.}$ hence evaluate : 6+4

$$\int_0^{\pi/4} (\cos 2\theta)^{3/2} \cos\theta \, d\theta$$

5. Attempt any *two* : 5 each

(*a*) Evaluate $\int \frac{dx}{x(x^2+1)^3}$

(*b*) $\int (3x - 2) \sqrt{x^2 + x + 1}^{dx}$.

(*c*) Prove that :

$$\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)dx$$

if $f(2a - x) = f(x)$.

$= 0$ if $f(2a - x) = -f(x)$

(*d*) Evaluate $\iint(x^2 + y^2)dxdy$ over the region in positive quadrant for which $x + y \leq 1$.