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RA—147—2022

FACULTY OF ARTS

B.A. (Third Year) (Fifth Semester) EXAMINATION

MAY/JUNE, 2022

(New Course)

MATHEMATICS

Paper XII

(Metric Spaces)

(Friday, 10-6-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—50

N.B. :— (*i*) Attempt *All* questions.

(*ii*) *All* questions carry equal marks.

1. In any metric space (X, d) prove that : 10

(*i*) The union of an arbitrary family of open sets in open.

(*ii*) The intersection of a finite number of open sets is open.

Or

Let A and B be any two subsets of a metric space (X, d), then prove that :

(*i*) $\overline{A}$ is a closed set

(*ii*) $A = \overline{A}$ if and only if A is closed.

2. Let (X, d) be a complete metric space and Y be a subspace of X. Then prove that Y is complete if and only if it is closed in (X, d). 10

Or

Let (X, d_1) and (Y, d_2) be any two metric spaces and f is a function from X into Y. Then prove that f is continuous at $a \in X$ if and only if for every sequence $\{a_n\}$ converging to 'a' we have $\lim_{n \to \infty} f(a_n) = f(a)$.

P.T.O.

3. Define compact subset. Prove that every closed subset of a compact space is compact. 10

Or

Prove that continuous image of a compact set is compact.

4. Prove that a subset Y of a metric space X is disconnected if and only if $Y \subseteq G_1 \cup G_2$ where, G_1 and G_2 are open sets in X such that : 10
$Y \cap G_1 \neq \phi, Y \cap G_2 \neq \phi, G_1 \cap G_2 \cap Y = \phi$.

Or

Prove that continuous image of a connected set is connected.

5. Attempt any *two* of the following : 5 each

(*a*) Prove that the function $d : \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ defined by $d(x, y) = |x - y|$ $\forall (x, y) \in \mathbf{R} \times \mathbf{R}$ is a metric on the set $\mathbf{R}$ of all real numbers.

(*b*) Let (X, d_1) and (Y, d_2) be metric spaces. Show that $f : x \to y$ is continuous if and only if $F(\bar{A}) \subseteq \overline{F(A)}$ for every $A \subseteq X$.

(*c*) Prove that the open interval]0, 1[with the usual metric is not compact.

(*d*) Let A be a connected subset of a metric space X and Let B be a subset of X such that $A \subseteq B \subseteq \bar{A}$. Then prove that B is also connected.