This question paper contains **2** printed pages]

RA—127—2022

FACULTY OF ARTS

B.A. (Third Year) (Sixth Semester) EXAMINATION

MAY/JUNE, 2022

(New Course)

MATHEMATICS

Paper–XVI

(Integral Transform)

(Thursday, 09-06-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) Using Laplace transforms, solve the differential equations : 15

$$(D + 1)y_1 + (D - 1)y_2 = e^{-t}$$

$$(D + 2)y_1 + (D + 1)y_2 = e^{t}$$

where $D = \frac{d}{dt}$ and $y_1(0) = 1,\ y_2(0) = 0$

Or

(*a*) Prove that Laplace transform of derivative of order n : 8

$$L[f^n(t] = S^nL[f(t)] - S^{n-1}f(0) - S^{n-2}f'(0) - S^{n-3}$$

$$f''(0) \ldots\ldots\ldots\ldots\ldots - f^{n-1}(0).$$

(*b*) Find the Fourier transform of : 7

$$f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

2. Find the inverse Laplace transform of $\frac{s+4}{s(s-1)(s^2+4)}$ 15

P.T.O.

Or

(*a*) Prove that Fourier complex integral : 8

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-iux}du \int_{-\infty}^{\infty} f(t)e^{iut}dt$$ 7

(*b*) Find the Fourier sine integral of

$$f(x) = e^{-i\ \beta x}\ (\beta > 0)$$

Hence show that :

$$\frac{\pi}{2}\ e^{-i\beta x} = \int_0^{\infty} \frac{\lambda \sin \lambda x}{\beta^2 + \lambda^2} d\lambda .$$

3. Attempt any *two* of the following :

(*a*) If F(s) is the complex Fourier transform of $f(x)$, then prove that : 5

$$F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

(*b*) Using the Laplace transform, find the solution of the initial value problem : 5

$$y'' + 25y = 10 \cos 5t$$

$$y(0) = 2,\ y'(0) = 0$$

(*c*) Find the inverse Laplace transform of $\frac{2s-5}{9s^2-25}$. 5

(*d*) If $L[F(t)] = F(s)$, then prove that $L\left[\frac{1}{t}f(t)\right] = \int_s^{\infty} F(s)ds$. 5