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RA—90—2022

FACULTY OF ARTS

B.A. (Third Year) (Sixth Semester) EXAMINATION

JUNE/JULY, 2022

(New Course)

MATHEMATICS

Paper XV

(Complex Analysis)

(Tuesday, 7-6-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time—2½ *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Suppose that $f(z) = u(x, y) + i\,v(x, y)$ and that $f'(z)$ exists at a point $z_0 = x_0 + iy$. Then show that the first order partial derivatives of u and v must exist at (x_0, y_0), and they must satisfy the Cauchy-Riemann equations $u_x = v_y$, $u_y = -v_x$ there. Also, prove that $f'(z_0)$ can be written $f'(z_0) = u_x + iv_x$ where these partial derivatives are to be evaluated at (x_0, y_0). 15

Or

(*a*) If a function f is analytic throughout a simply connected domain D, then prove that $\int_C f(z)\,dz = 0$ for every closed contour C lying in D. 8

P.T.O.

(*b*) If $f(z)=\frac{z}{\bar{z}}$, then show that the limit $\lim_{z\to 0} f(z)$ does not exist. 7

2. Suppose that a function $f(z)$ is continuous on a domain D. If any one of the following statements is true then prove that others are true : 15

(*i*) $f(z)$ has an antiderivatives F(z) in D.

(*ii*) The integral of $f(z)$ along contours lying entirely in D and extending from any fixed point z_1 to any fixed point z_2 all have the same value.

(*iii*) The integrals of $f(z)$ around closed contours lying entirely in D all have value zero.

Or

(*a*) Describe the method to find roots of complex numbers. Also find the values of $(-8i)^{1/3}$. 8

(*b*) (Fundamental Theorem of Algebra) Prove that any polynomial

$$p(z) = a_0 + a_1 z + a_2 z^2 + \ldots\ldots + a_n z^n \ (a_n \neq 0)$$

of degree n ($n \geq 1$) has at least one zero. 7

3. Attempt any *two* of the following : 5 marks each

(*a*) If a function $f(z)$ is continuous and non-zero at a point z_0, then prove that $f(z) \neq 0$ throughout some neighborhood of that point.

(*b*) Find the harmonic conjugate $v(x, y)$ of a function :

$$u(x, y) = y^3 - 3x^2 y.$$

(*c*) If C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$, then show that :

$$\left| \int_C \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7} .$$

(*d*) Show that $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$ whenever $|z| < 1$.