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RA—200—2022

FACULTY OF ARTS

B.A. (Third Year) (Fifth Semester) EXAMINATION

MAY/JUNE, 2022

(New Course)

MATHEMATICS

Paper XIII (DSE-IB)

(Linear Algebra)

(Monday, 13-6-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—50

N.B. :— (*i*) Attempt *All* questions.

(*ii*) *All* questions carry equal marks.

1. Define direct sum : 10

Let U and W be two subspaces of a vector space V and Z = U + W. Then prove that $Z = U \oplus W$ if and only if the following condition is satisfied. Any vector $z \in Z$ can be expressed uniquely as the sum

$$z = u + w,\ u \in U,\ w \in W$$

Or

Give the definition of basis for a vector space V, and prove that in a vector space V if $\{v_1, v_2, ...v_n\}$ generates V an if $\{w_1, w_2 ..... w_n\}$ is linearly independent then $m \leq n$.

P.T.O.

2. If U and W are two subspaces of a finite dimensional vector space V, then show that :

$$\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$$

Or

Let T : U $\to$ V be linear map. Then show that :

(*a*) R(T) is a subspace of V.

(*b*) N(T) is a subspace of V.

(*c*) T is one-one iff N(T) is the zero subspace $\{O_u\}$ of U.

(*d*) If $[u_1, u_2 \ldots u_n] = U$, then $R(T) = [T(u_1), T(u_2), \ldots\ldots T(u_n)]$

(*e*) If U is finite dimensional then

$$\dim R(T) \le \dim U.$$

3. State and prove Rank-Nullity Theorem. 10

Or

Let T : U $\to$ V be a non-singular linear map. Then prove that :

$T^{-1} : V \to U$ is a linear one-one and onto map.

4. The dimension of the vector space $M_{m,n}$ is $m.n.$ 10

Or

Let A be a square matrix of order n having k distinct eigen values $\lambda_1, \lambda_2 \ldots \lambda_k$. Let v_i be an eigen vector corresponding to the eigen values λ_i, $i = 1, 2, \ldots, k$. Then show that the set $\{v_1, v_2 \ldots v_k\}$ is linearly independent.

5. Attempt any *two* of the following : 5 each

(*a*) Check the linear dependence or linear independence of the set $\{x, |x|\}$ is G(–1, 1).

(*b*) Check whether the following map is linear or not.

$T : V_3 \to V_2$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3)$

(*c*) Consider the mapping $T : P_2 \to V_3$ defined by $T(\alpha_0 + \alpha_1 x + \alpha_2 x^2) = (\alpha_0, \alpha_1, \alpha_2)$, check T is a linear map and check when T^{-1} exists or not.

(*d*) Determine the matrix associated with a linear map :

$T : V_3 \to V_3$ defined by

$T(x_1, x_2, x_3) = (x_1 - x_2 + x_3, 2x_2 + 3x_2 - \frac{1}{2}x_3, x_1 + x_2 - 2x_3)$ related to the bases.

$B_1 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

$B_2 = \{(1, 1, 0), (1, 2, 3), (-1, 0, 1)\}$