SUBJECT CODE NO:- SB-69
FACULTY OF SCIENCE
B. Sc (Third year) (Sem-VI)
Examination JUNE/JULY 2022
Mathematics
Integral Transforms-XVI

[Time: 2:00 Hours] **[Max. Marks: 40]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 If $L[f(t)] = F(s)$ then prove that $L[t^n.f(t)] = (-1)^n.\frac{d^n}{ds^n}[F(s)]$ and hence find the Laplace transform of t. sinhat 15

OR

a. Find the value of $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$ 08

b. Find the inverse Laplace transform of $\frac{1}{s(s+a)}$ 07

Q.2 Prove that the Fourier integral for $f(x) = \frac{1}{\pi}\int_0^\infty \int_{-\infty}^\infty f(t).cosu(t-x)dtdu$ 15

OR

a) Using Laplace transforms find the solution of the initial value problem. 08

$$y'' - 4y' + 4y = 64sin2t$$
$$y(0) = 0, y'(0) = 1$$

b) Using Laplace transforms find the solution of the initial value problem 07

$y'' + 25y = 10\cos 5t$
$y(0) = 2 \quad y'(0) = 0$

Q.3 Attempt any two of the following : 10

a) Prove that $L(e^{at}) = \frac{1}{s-a}$ where s >a.

b) Find the inverse Laplace transform of $\frac{8}{s^2-16}$

c) Prove that
$f(x) = \frac{2}{\pi}\int_0^\infty sinuxdu.\int_0^\infty f(t).sinutdt$ is the Fourier sine integral for $f(x)$

d) Solve $\frac{dx}{dt} + y = 0$ and $\frac{dy}{dt} - x = 0$ under the condition. $x(0) = 1, y(0) = 0.$